Ensembles of Chaotic Populations Yield More Than Just Chaos

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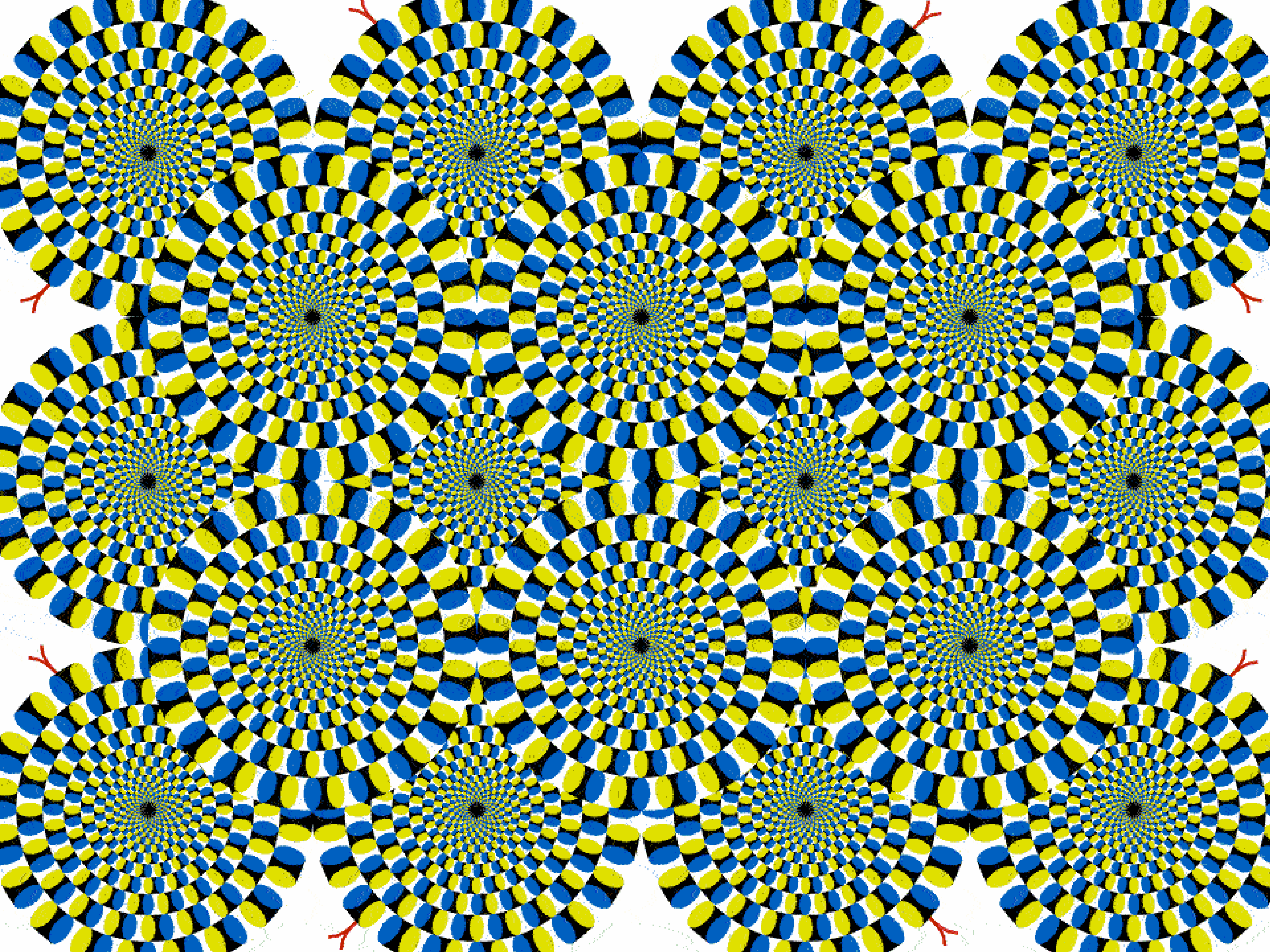
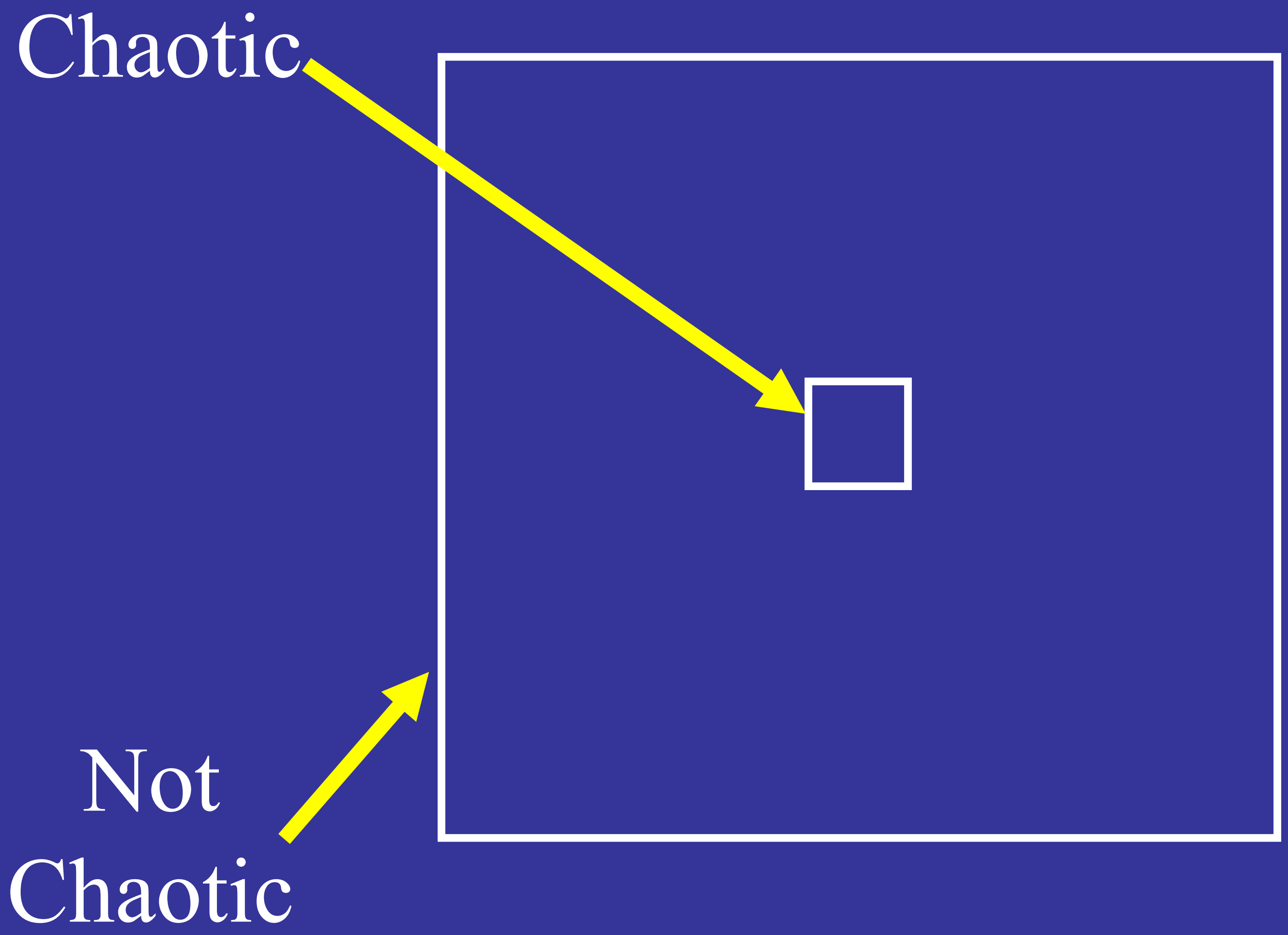


"The intensive '**search for holy chaos**' … was a failure, because so far we have not found any direct analogs of chaotic … dynamics in nature."

Turchin, P. 2003. Complex population dynamics.

Perhaps this failure is a matter of scale



Population growth under controlled conditions often follows the logistic model

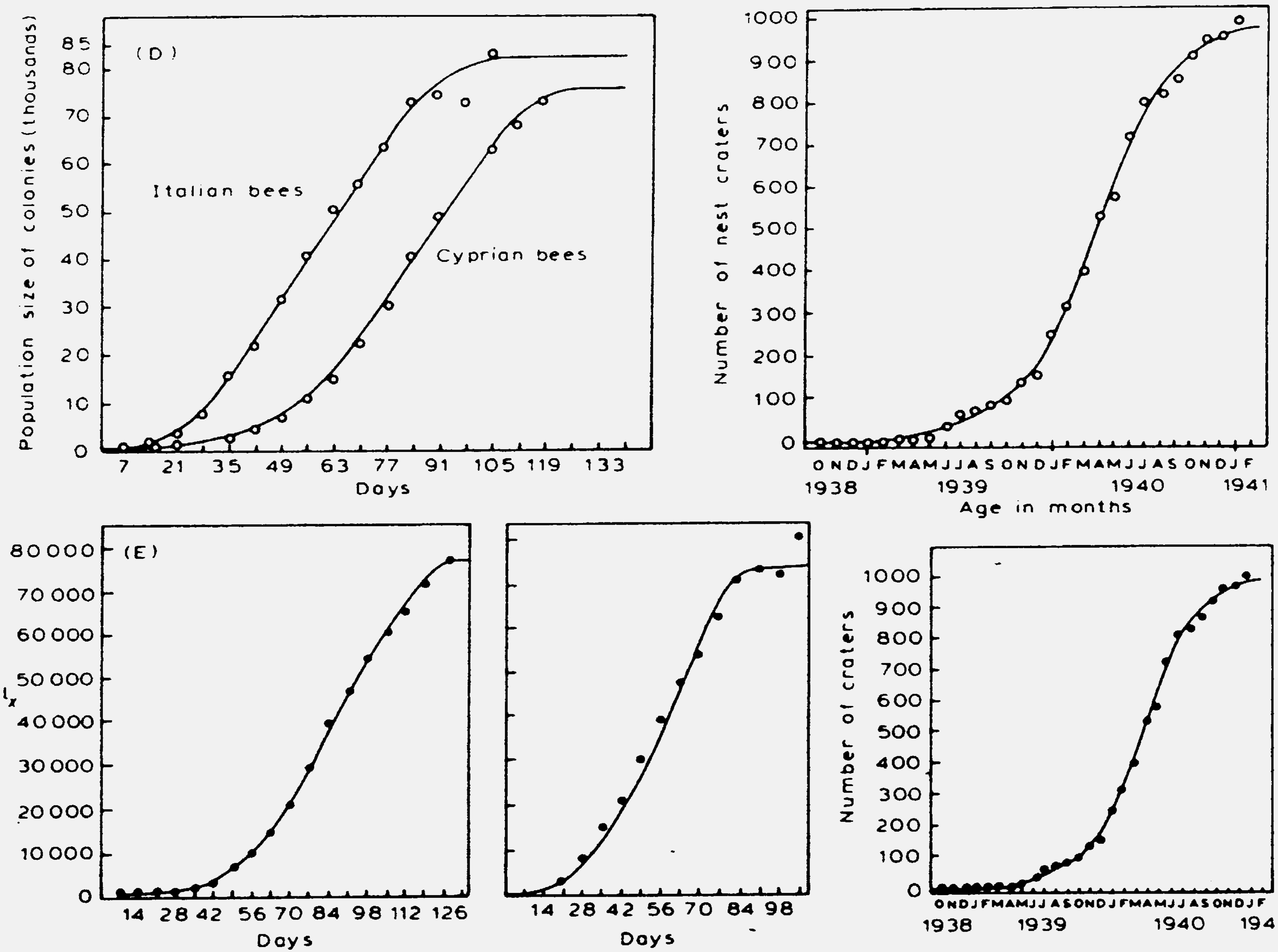
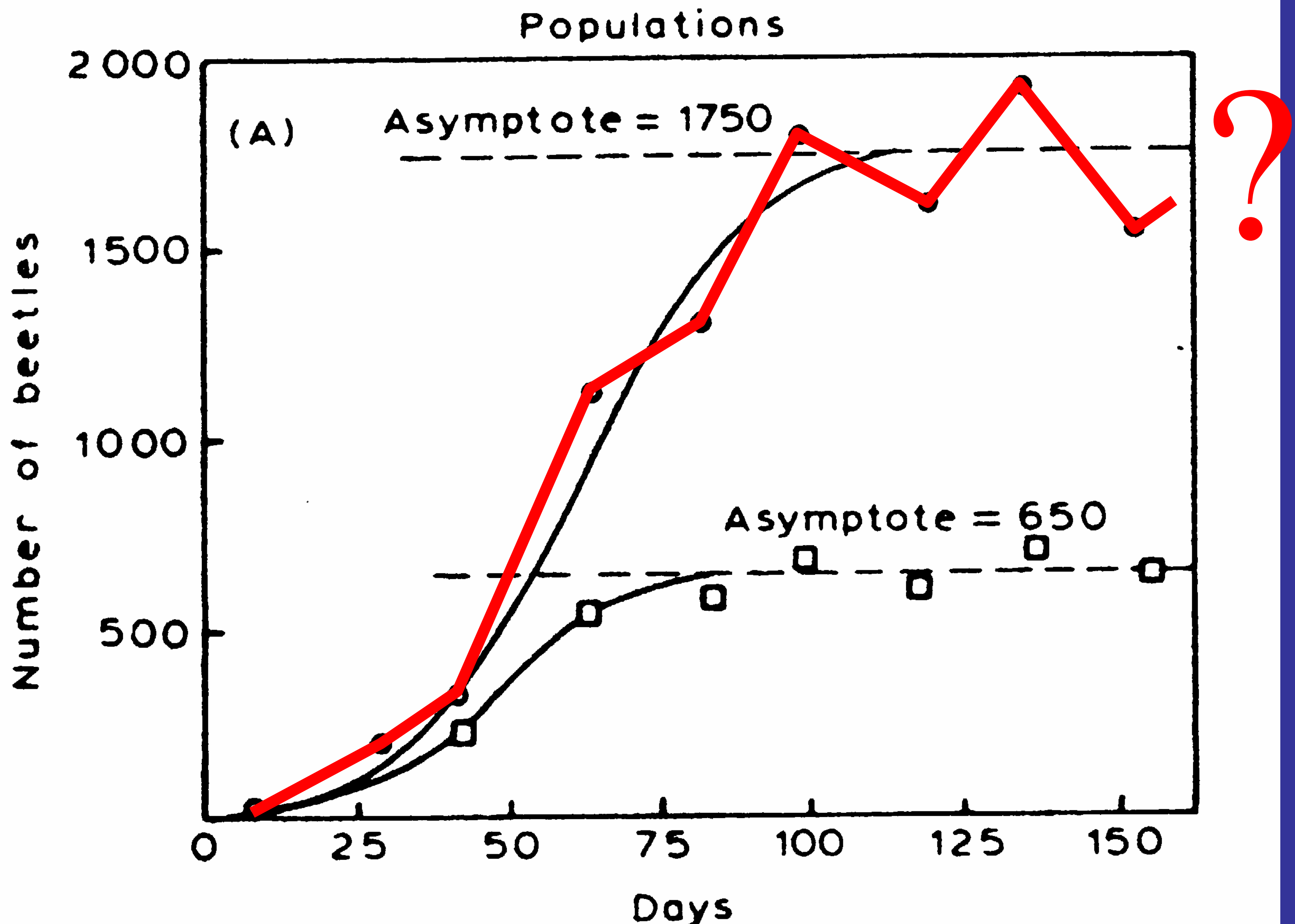
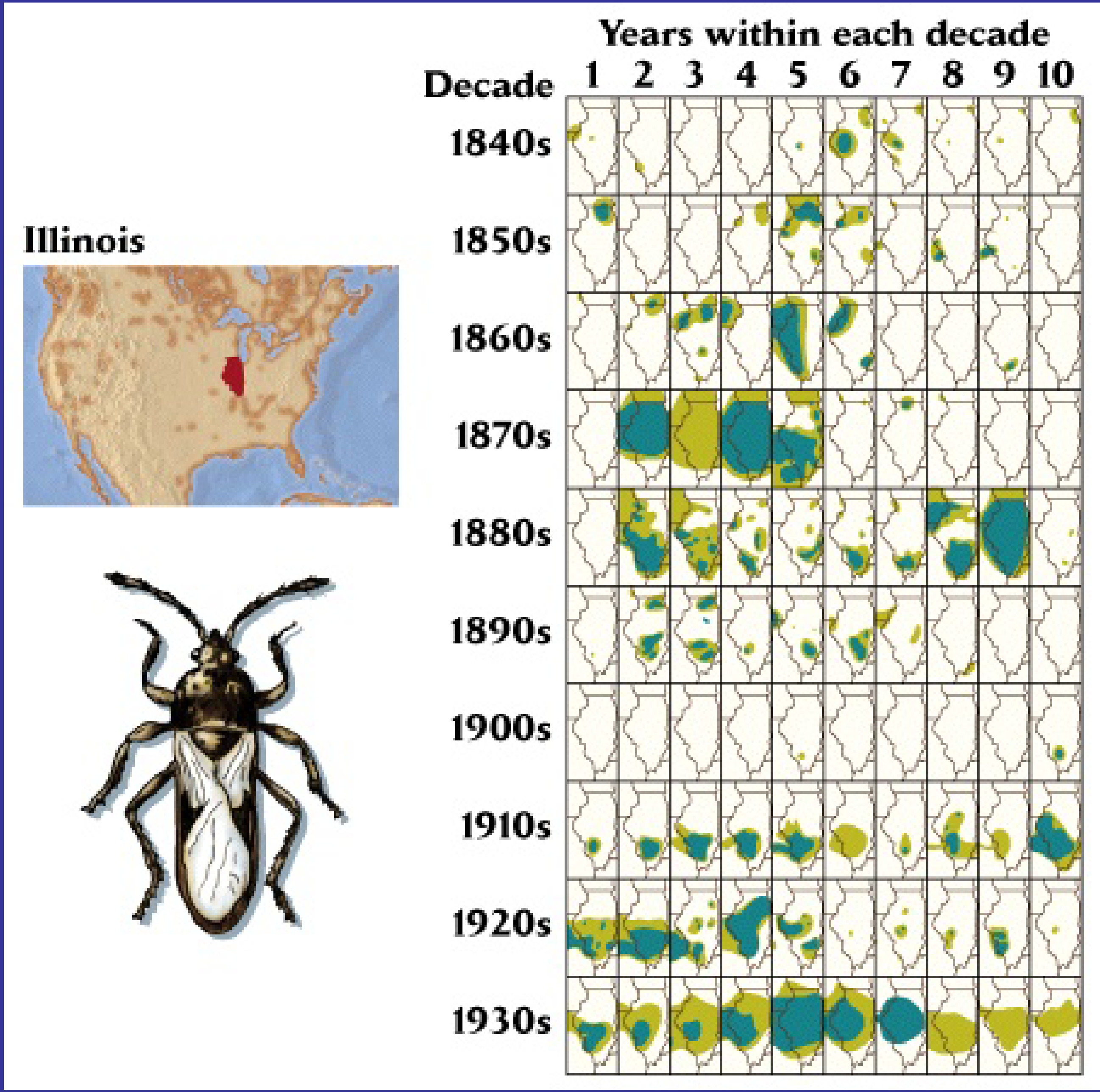


Fig. 3 (continued). (E) The same information as (D) as presented in Hutchinson, 1978. Note that in no cases in (D) or (E) does the logistic fit the final data points given better than a linear extrapolation. (With permission from Yale University Press, New Haven, CT.).



Populations
(A)
Asymptote = 1750
Asymptote = 650
Number of beetles
Days
2000
1500
1000
500
0
25
50
75
100
125
150
?

Chinch bug



Ricklefs, Economy of Nature, 2000. Figure 13.16

What is chaos in population dynamics?

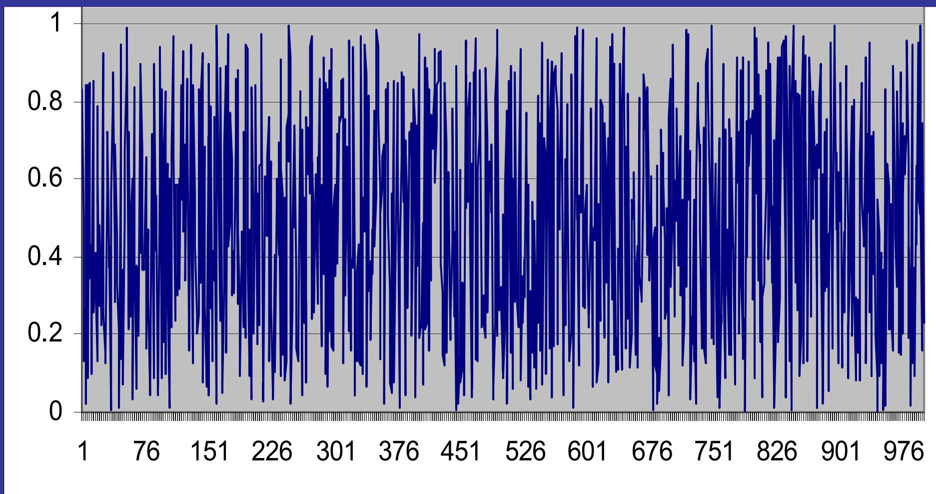
Signature of chaos = sensitivity to initial conditions

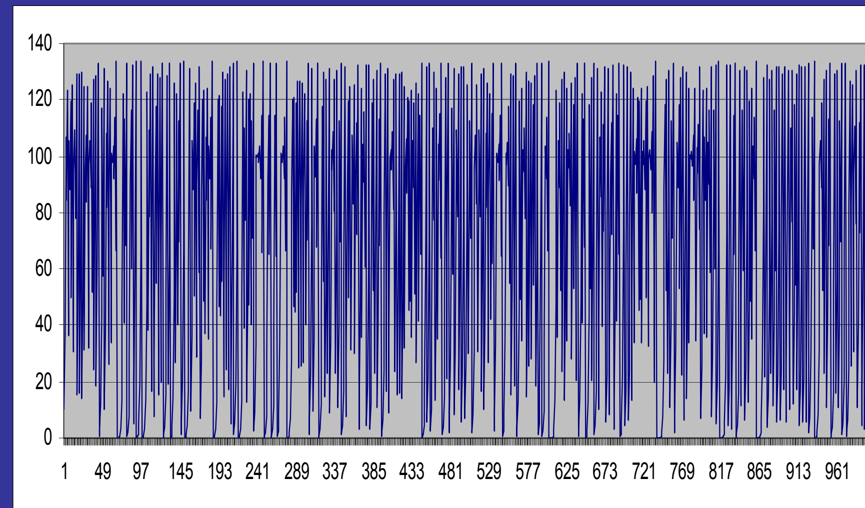
Assessed with:

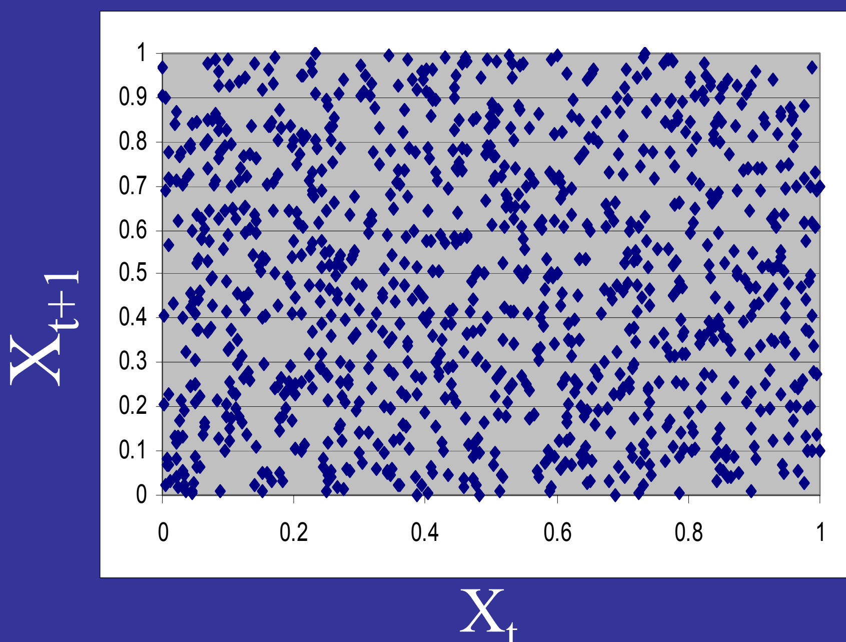
1. Pattern in N_{t+1} vs. N_t

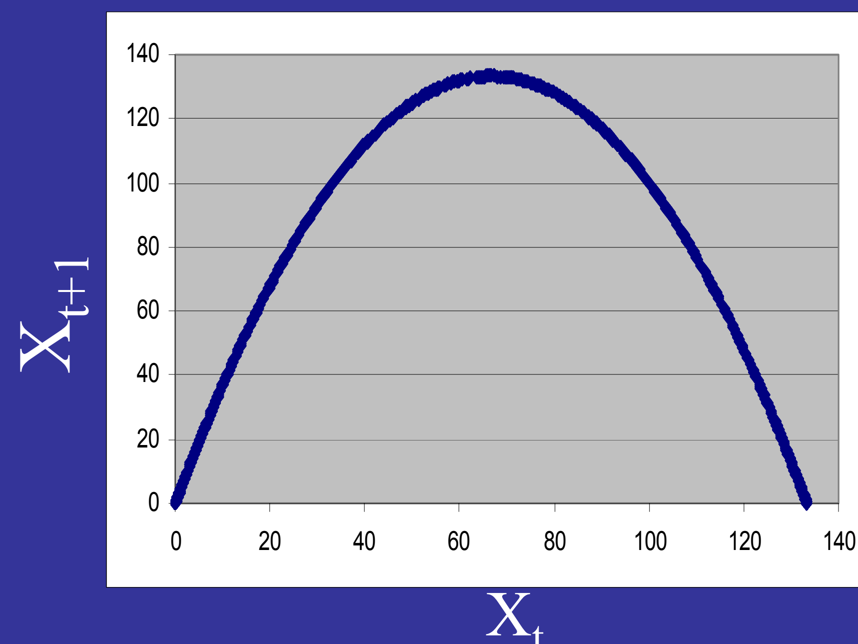
Random

Logistic









What is chaos in population dynamics?

Signature of chaos = sensitivity to initial conditions

Assessed with:

1. Pattern in N_{t+1} vs. N_t
2. Lyapunov exponent (λ)

chaos when $\lambda > 0$

Larch Bud Moth (Large-Scale Sampling)

TABLE 9.1. LBM primary data: summary of results of nonlinear time-series analysis. Quantities: number of data points, n; measure of amplitude (SD of log-transformed densities), S; dominant period, T; the autocorrelation at the dominant period, ACF[T]; estimated process order, p; polynomial degree, q; the coefficient of prediction of the best model, R^2_{pred}; and the estimated dominant Lyapunov exponent, Λ_∞

Location	n	S	T	ACF[T]	p	q	R^2_{pred}	Λ_∞
Sils	42	1.35	9	0.67**	3	2	0.79	0.05
Engadine	31	1.53	9	0.79**	2	2	0.91	−0.01
Briançonnais	20	1.25	8	0.68**	3	2	0.94	0.45
Goms	21	1.28	9	0.90**	2	2	0.94	0.23
Val Aurina	20	1.08	10	0.63**	2	1	0.83	−0.06
Lungau	19	1.15	9	0.76**	3	1	0.85	−0.01

Turchin 2003. Complex Population Dynamics

Lemmings (Small-Scale Sampling)

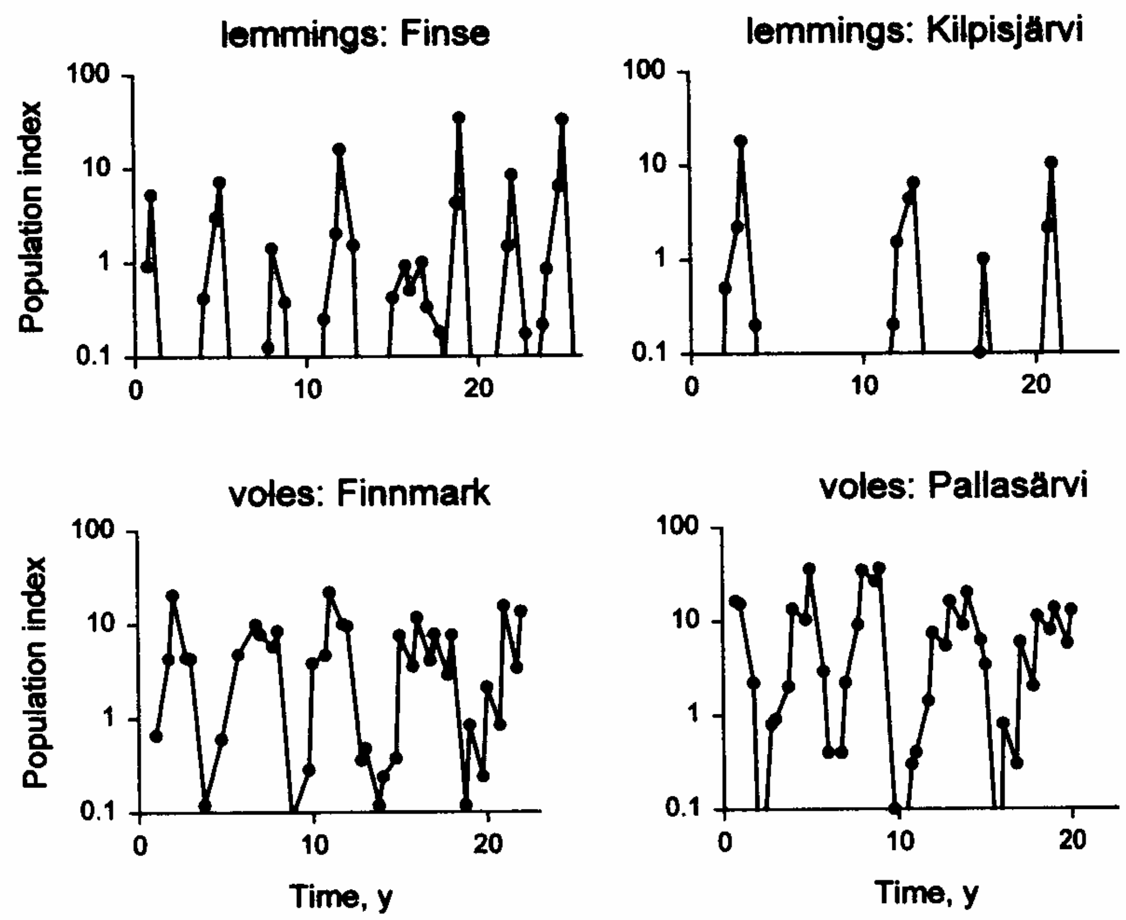


FIGURE 12.5. Population trajectories of lemmings in Finse and Kilpisjärvi; and voles in Finnmark and Pallasjärvi.

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Lemmings (Small-Scale Sampling)

TABLE 12.5. Summary of NLTSM results: lemmings. Same notation as in table 12.1

Location	*n*	*S*	*T*	ACF[*T*]	*p*	*q*	R^2_{pred}	Λ
L. lemmus								
Finse	25	1.25	3	0.37*	2	2	0.43	2.01
Kilpisjärvi	25	0.92	—	—	3	2	0.31	1.01
Finnmark	20	1.06	—	—	3	1	0.21	0.08
L. sibericus								
Barrow	18	0.77	—	—	2	2	0.35	0.53
Kolyma	12	0.79	4	0.85**	2	1	0.49	0.13
Indirect indices								
Pt. Barrow	21	0.73	3	0.43*	2	1	0.50	0.05
Taimyr	39	0.97	3	0.56**	3	1	0.32	0.34

Data sources: Finse (Framstad et al. 1997); Kilpisjärvi (Turchin et al. 2000, data collected by H. Henttonen); Finnmark (Ekerholm et al. 2000); Barrow (Pitelka 1976); Kolyma (Potapov 1997); Pt. Barrow (Schultz 1969); Taimyr (*Wadden Sea Newsletter*, 1997).

Series that were detrended: none.

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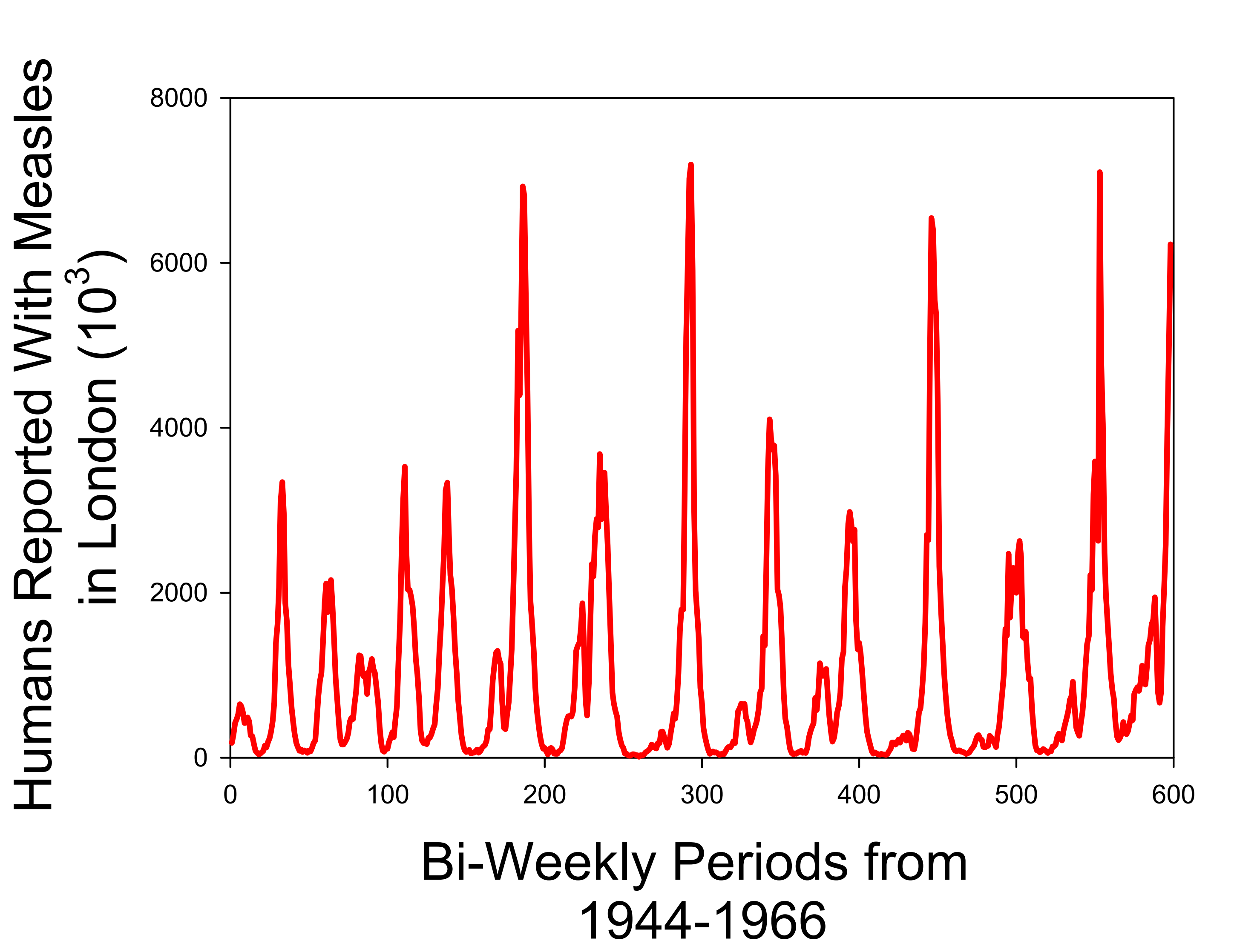
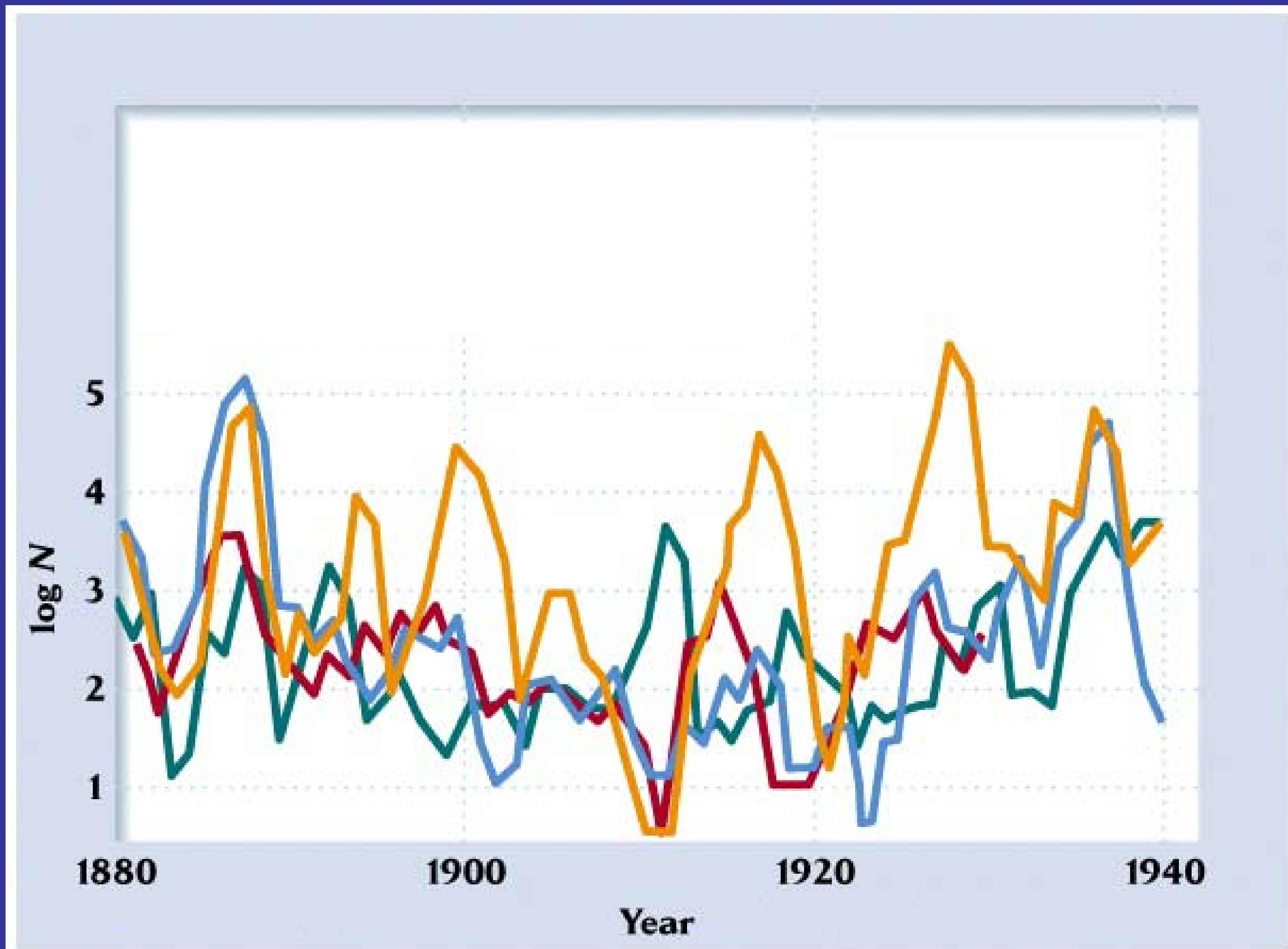
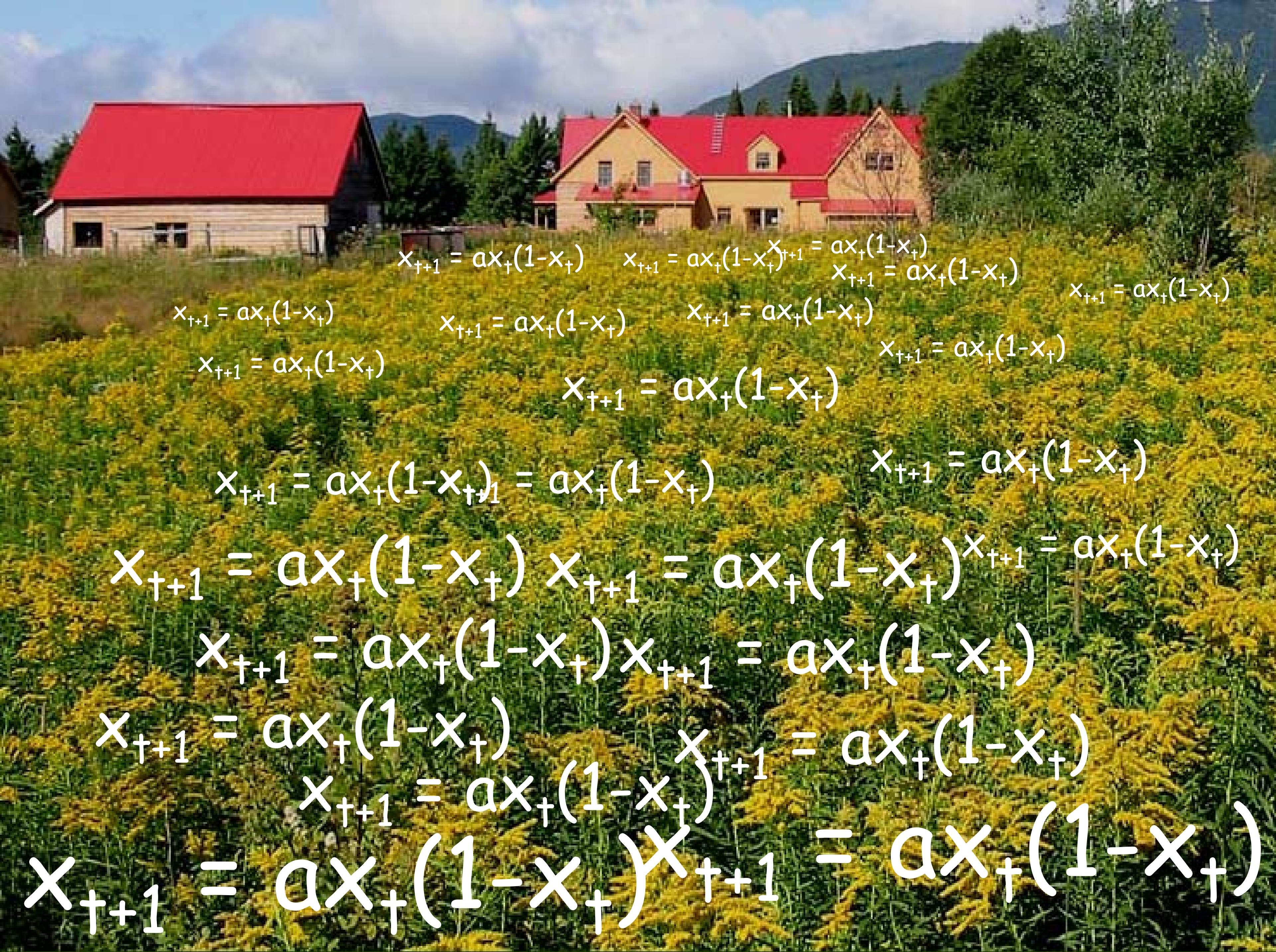
Humans With Measles (Small-Scale Sampling)



Data courtesy of Bryan Grenfell

Log # pupae or larvae during winter




$x_{t+1} = ax_t(1-x_t)$
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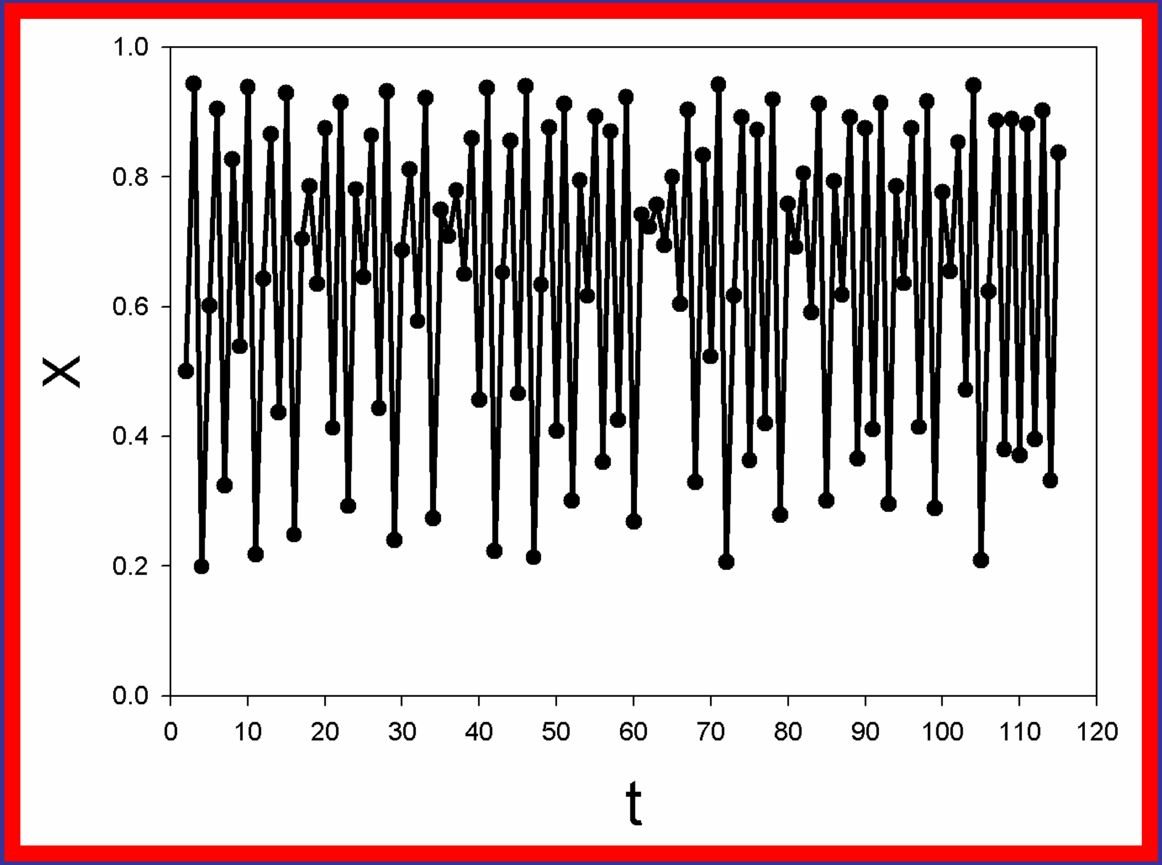
Example: $x_{t+1} = ax_t(1-x_t)$

$a = 3.77695$

1	0.5000
2	0.9442
3	0.1989
4	0.6017
5	0.9051
6	0.3243
7	0.8276
8	0.5388
9	0.9385
10	0.2178
11	0.6435
12	0.8664
13	0.4371
14	0.9293
15	0.2481
16	0.7046
17	0.7861
18	0.6352
19	0.8752
20	0.4125
21	0.9153
22	0.2928
23	0.7821



Example: $x_{t+1} = ax_t(1-x_t)$
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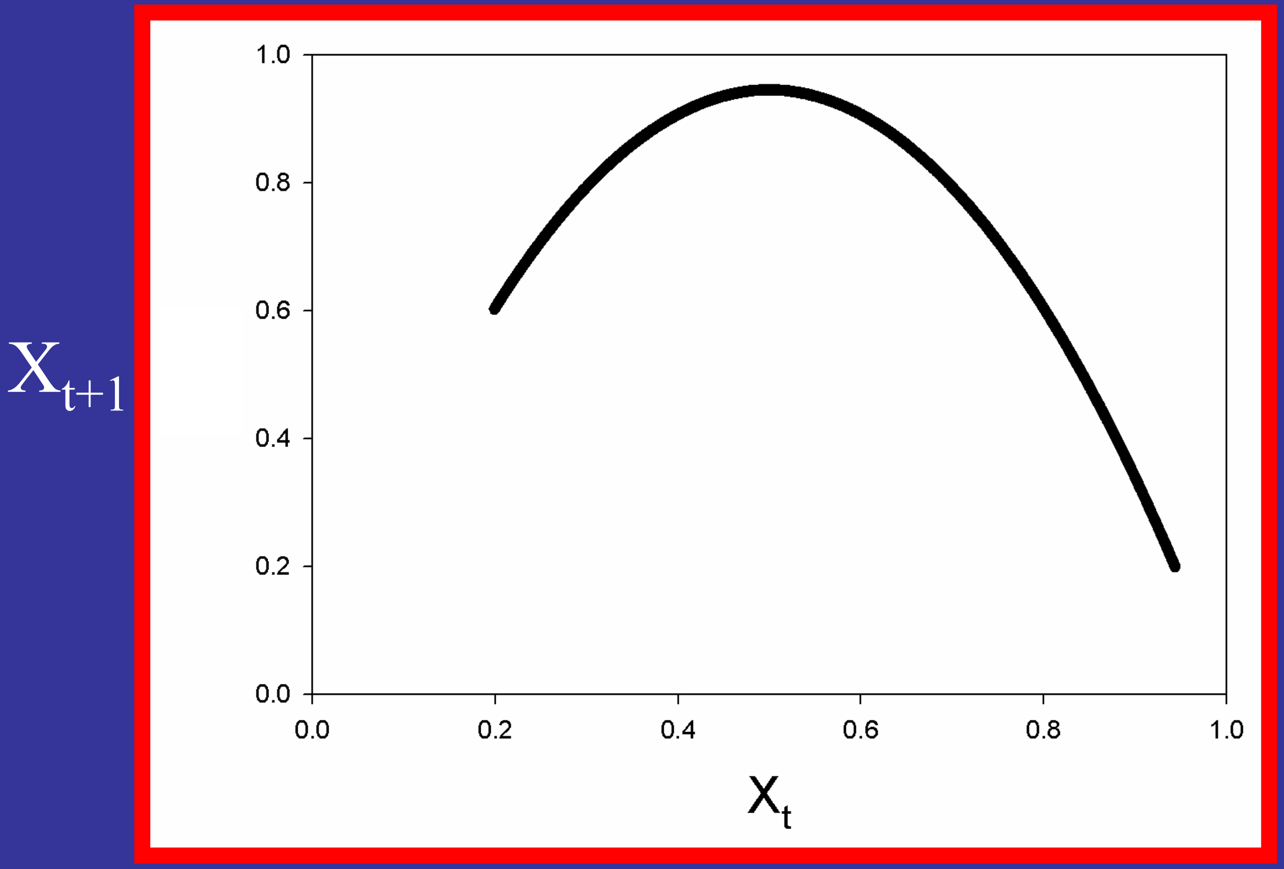
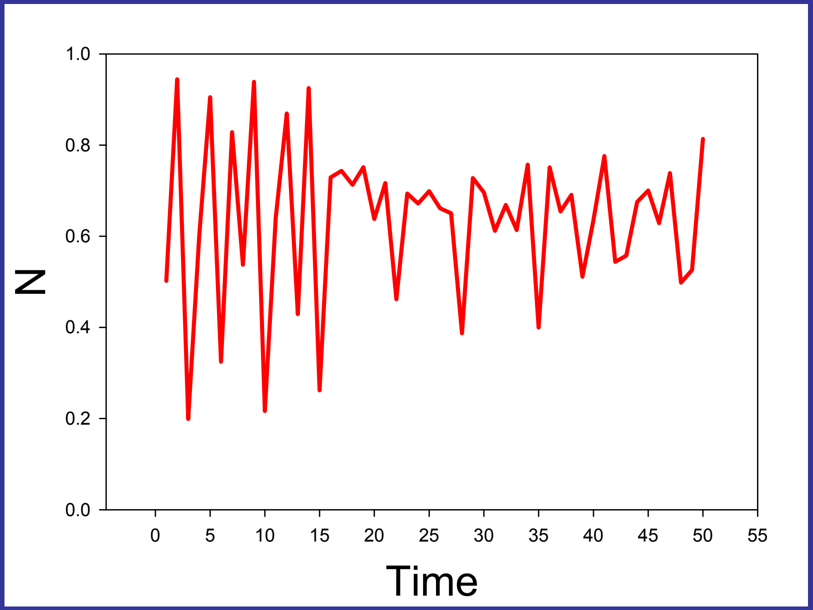
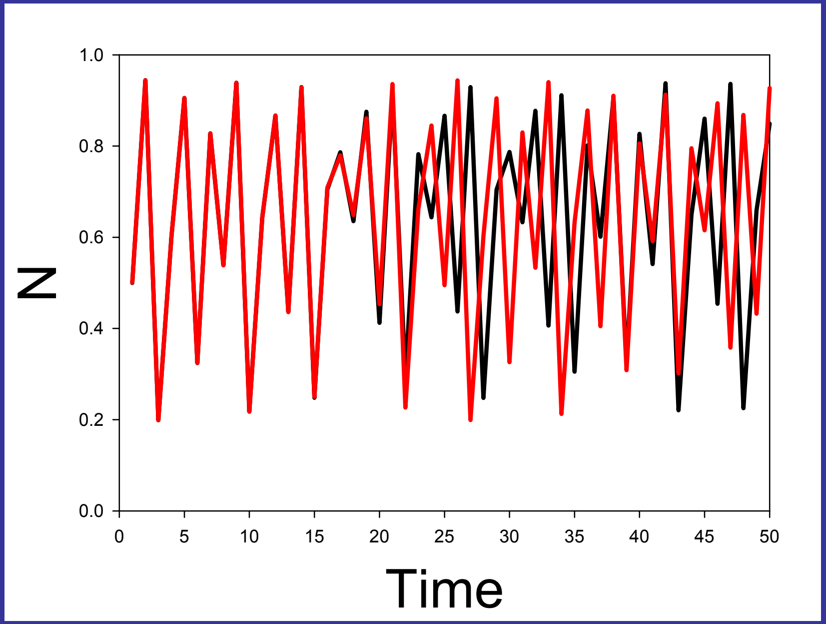


Example: $x_{t+1} = ax_t(1-x_t)$

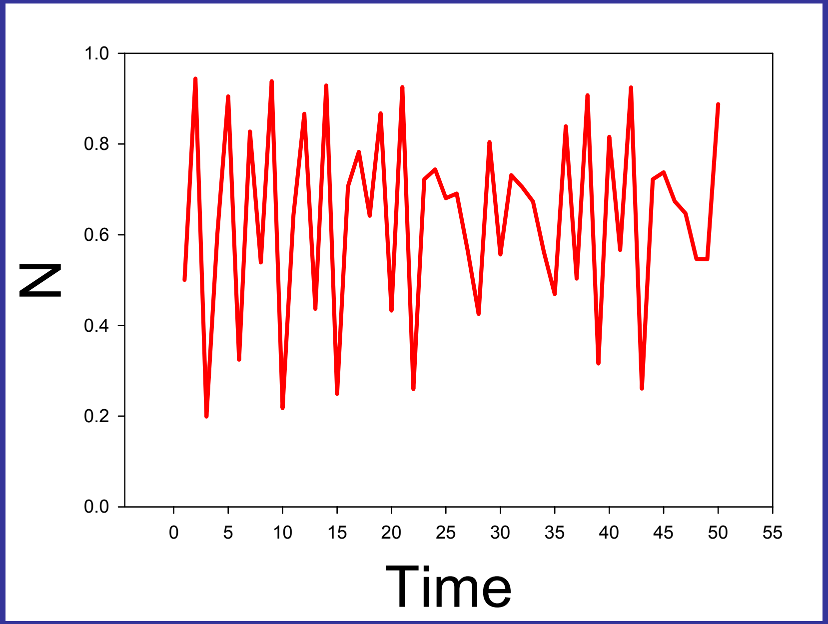
$a = 3.77695$

Average of Five Populations





Average

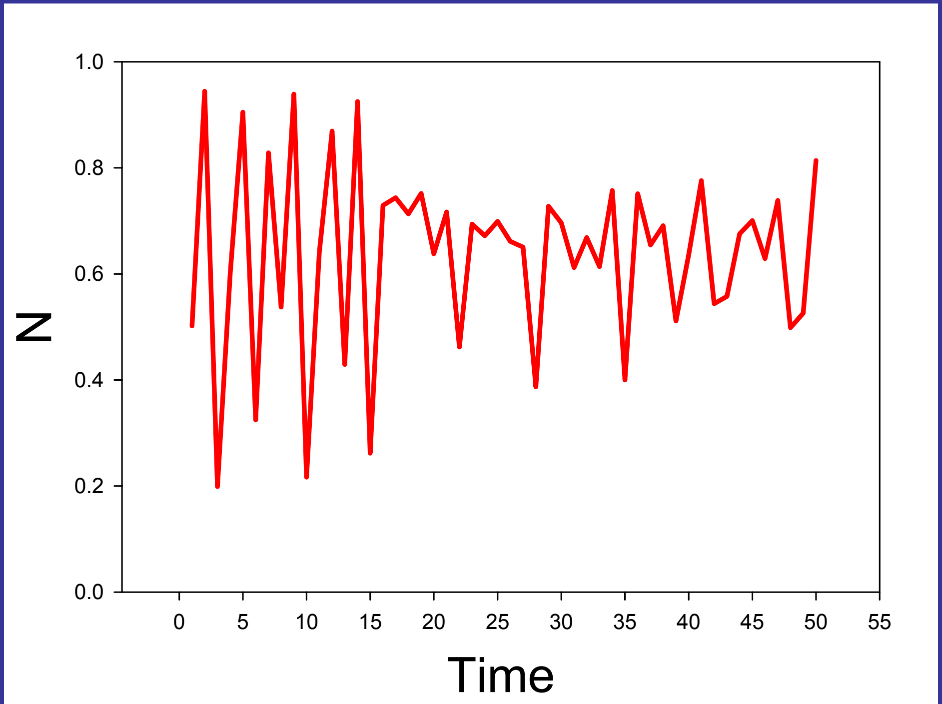


The 7-Step Approach

1. Collect 20K values of *a* that yield chaos.
2. Collect another 20K values of *a* that do not yield chaos but are in the same range ($3.57 \leq a < 4.00$) (“control”).
3. Run many simulations (10K) with **slightly different initial conditions** for each of the 40K values of *a*.

The 7-Step Approach

4. Run the simulations for 5000 time steps to avoid “transient effects.”

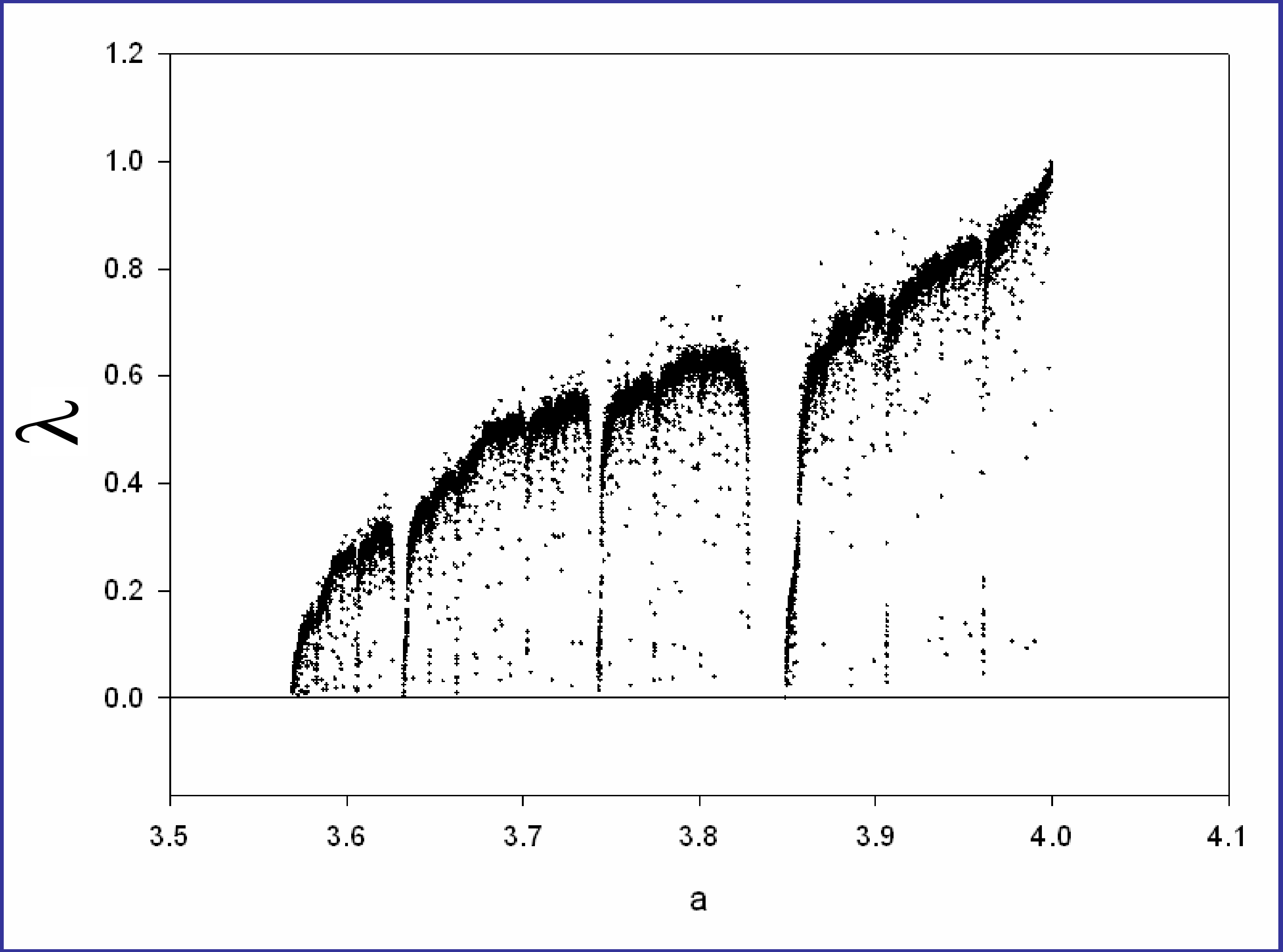


The 7-Step Approach

4. Run the simulations for 5000 time steps to avoid "transient effects."
5. Average all 10K sims with the same value of *a* for the last 5000 time steps.
6. Calculate the Lyapunov exponent (λ) for these average time series.
7. Plot λ against all values of *a*.

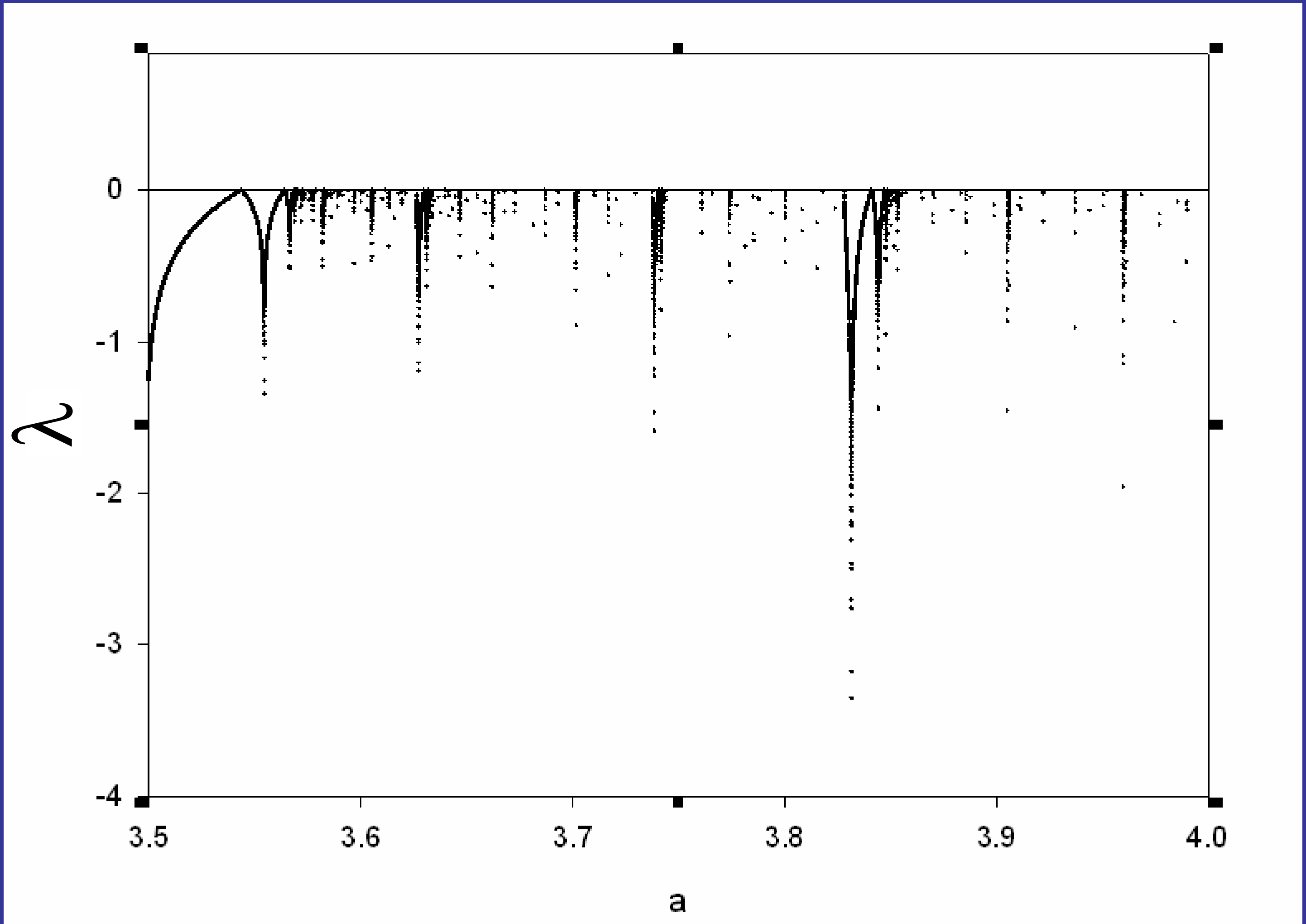
Single Populations: *a* is "chaotic"

λ for populations for time steps 5K -> 10K

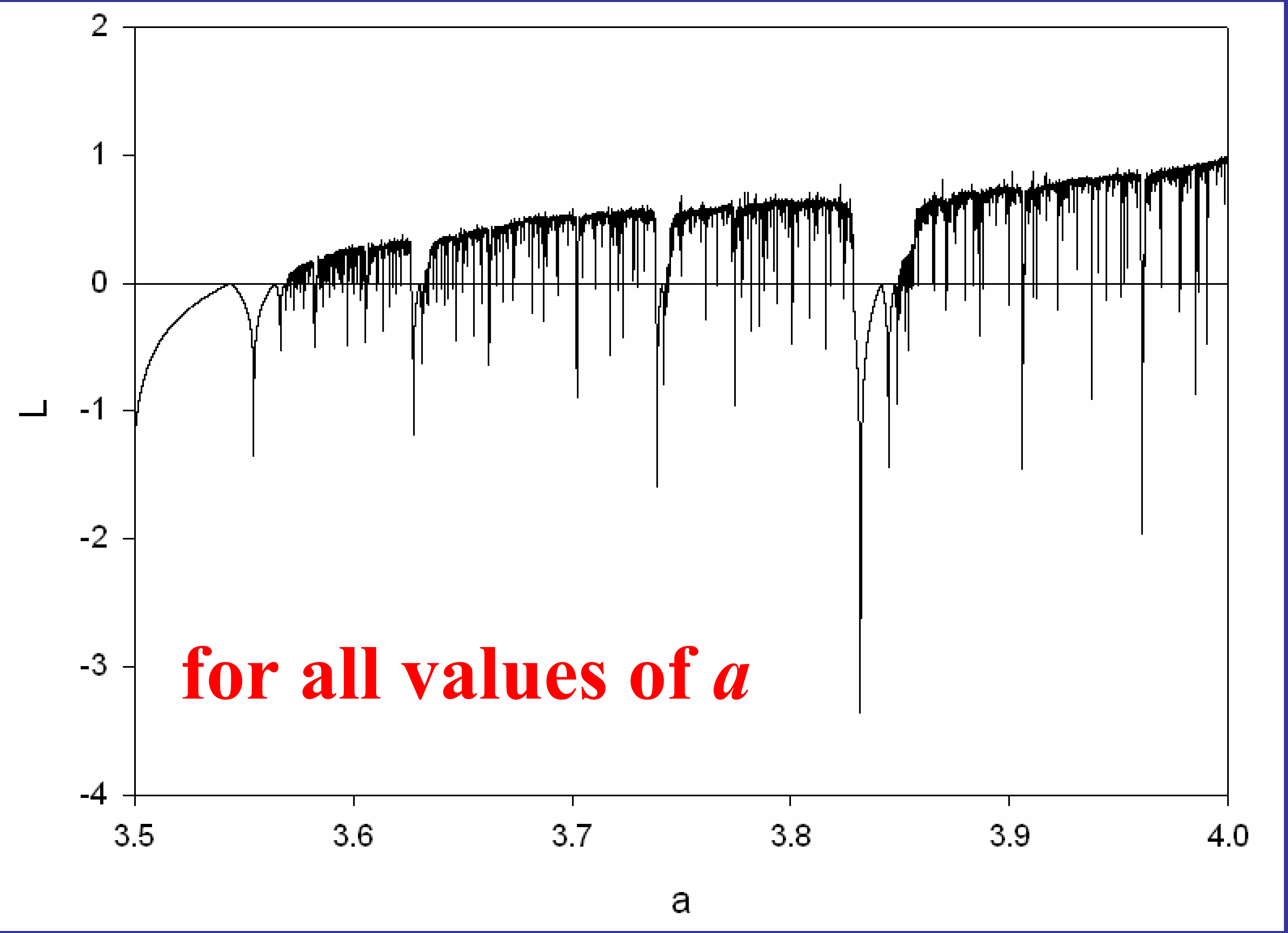


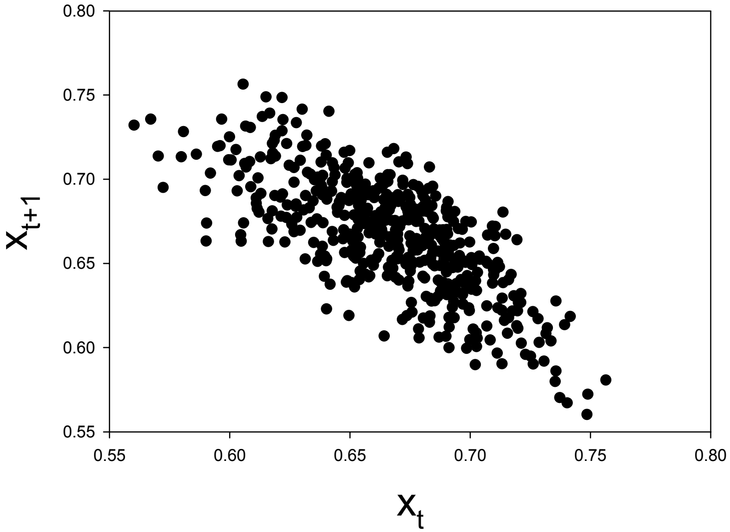
Single Populations: *a* is not “chaotic”

λ for populations for time steps 5K -> 10K



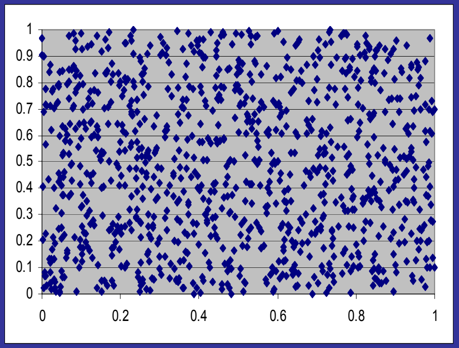
What do the average dynamics look like?



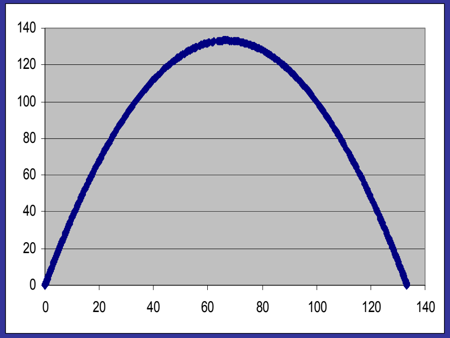


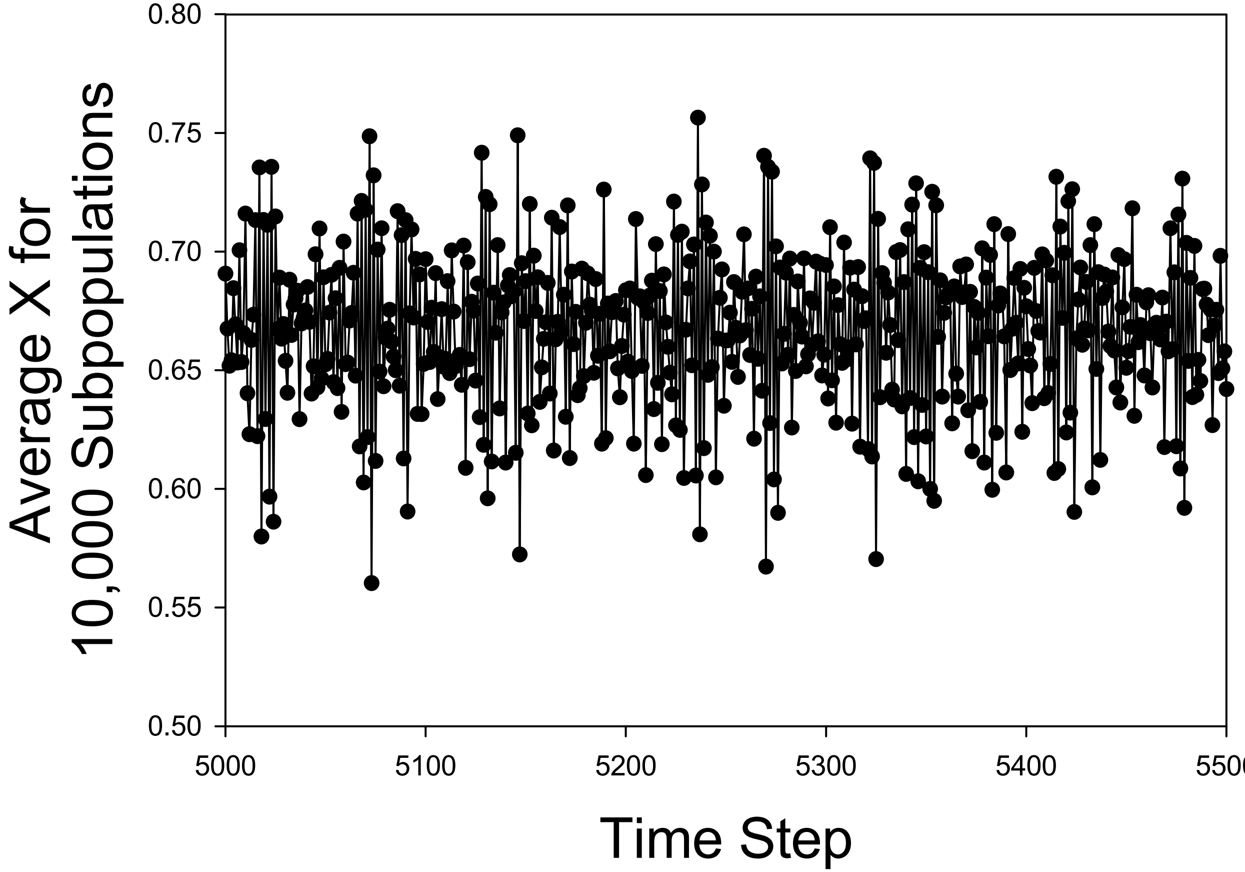
$$a = 3.70748$$
$$\lambda = 0.273$$

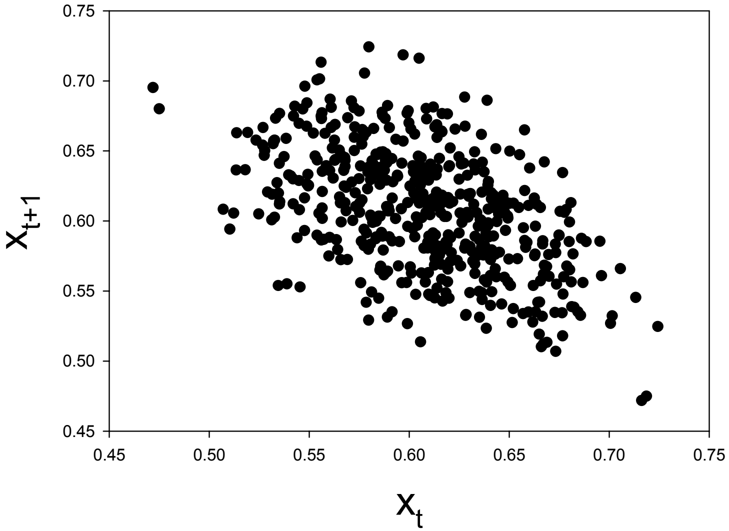
Random



Chaotic



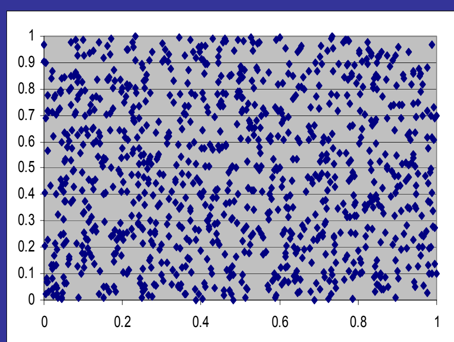




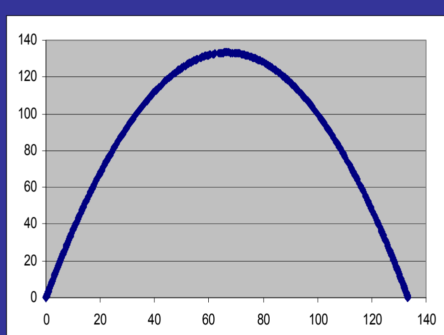
$a = 3.896664$

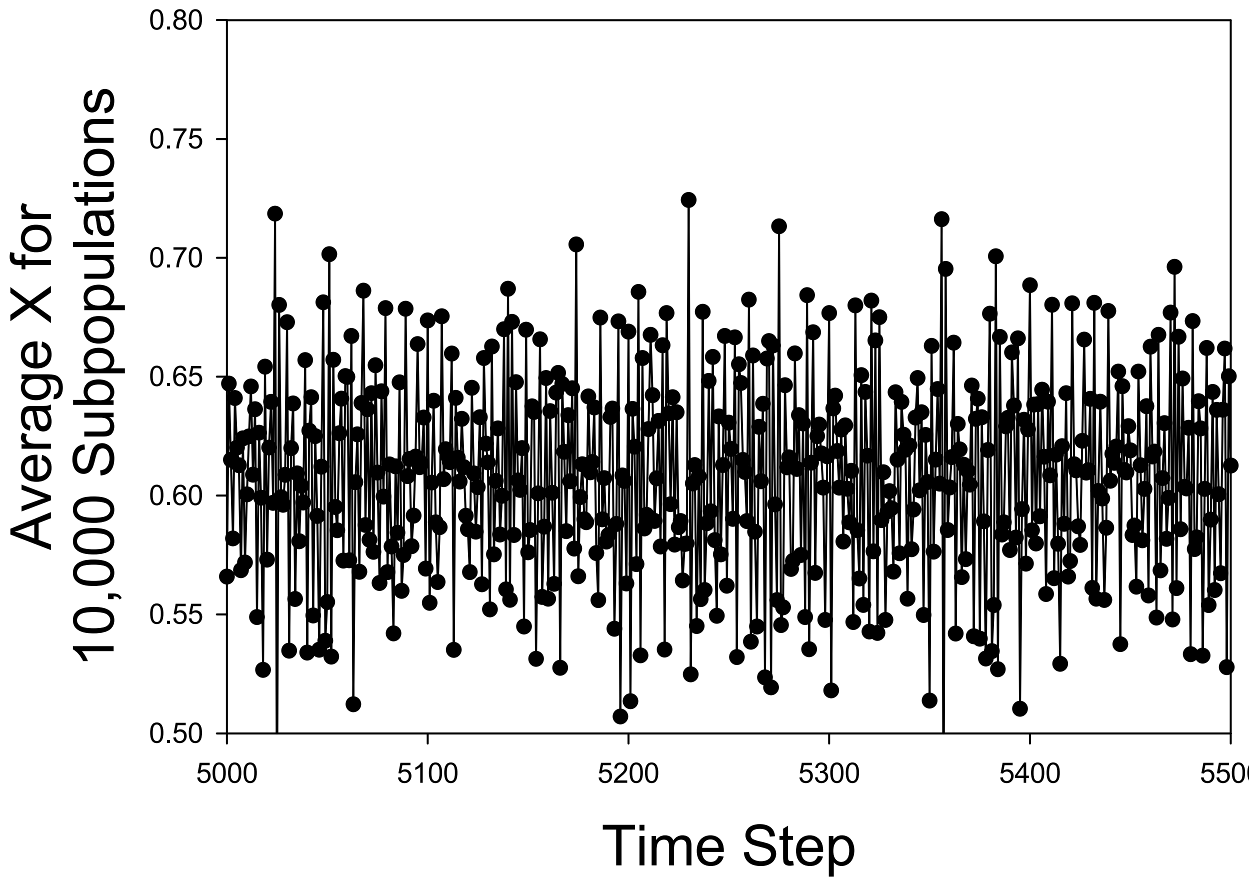
$\lambda = -0.444$

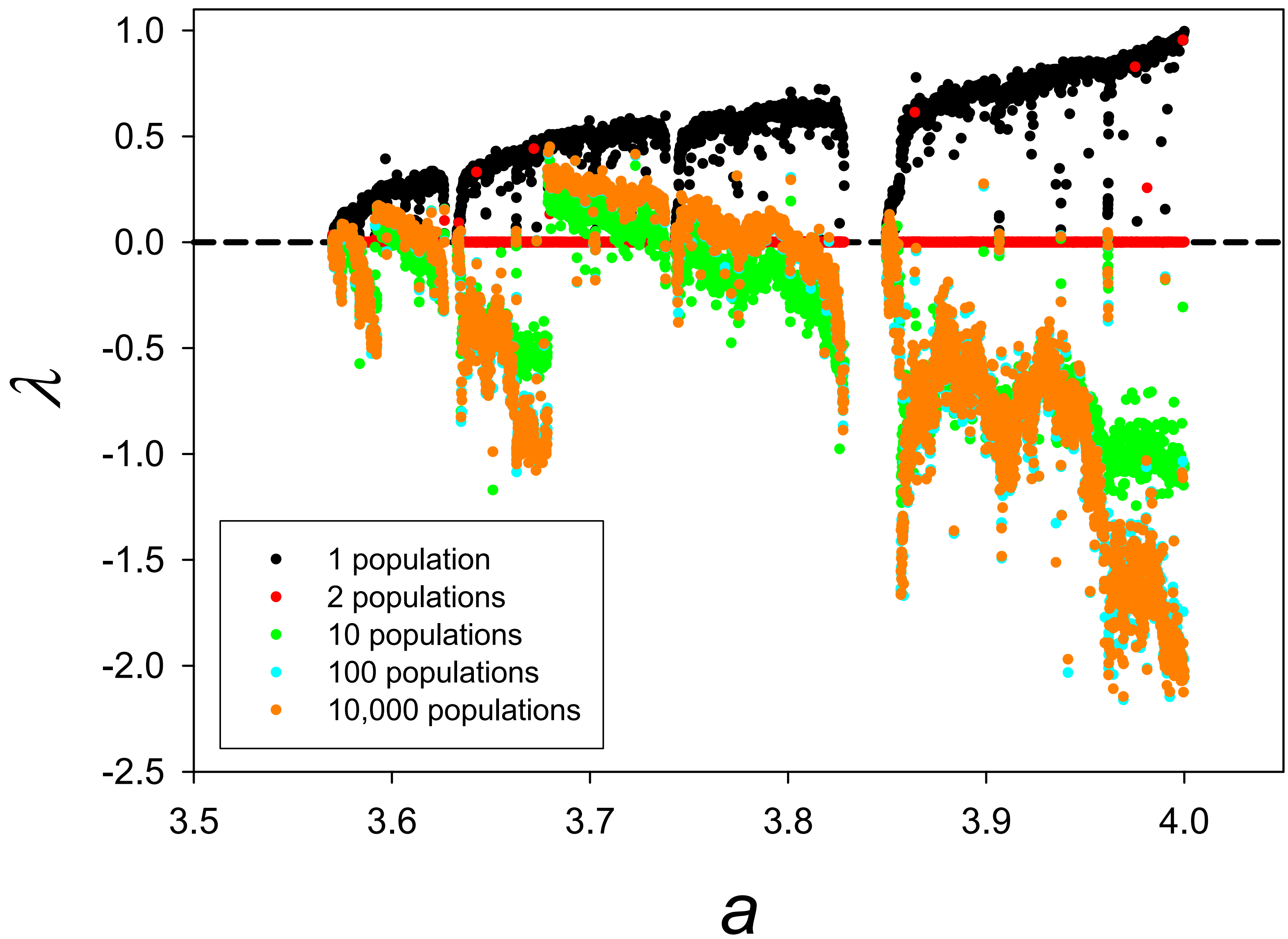
Random



Chaotic



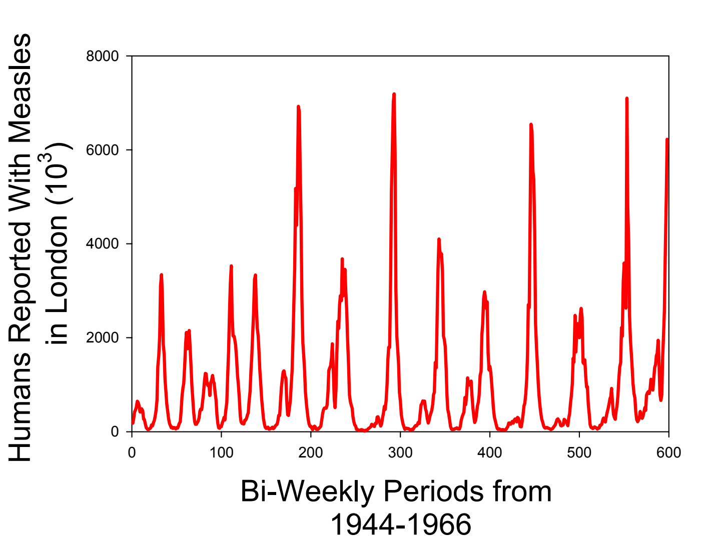




1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
λ
3.5
3.6
3.7
3.8
3.9
4.0
a
1 population
2 populations
10 populations
100 populations
10,000 populations

Conclusions (3)

1. Chaos has been detected occasionally in relatively big species (lemmings, grouse, and measles **in humans**) and far less convincingly in small-bodies species (e.g., insects).



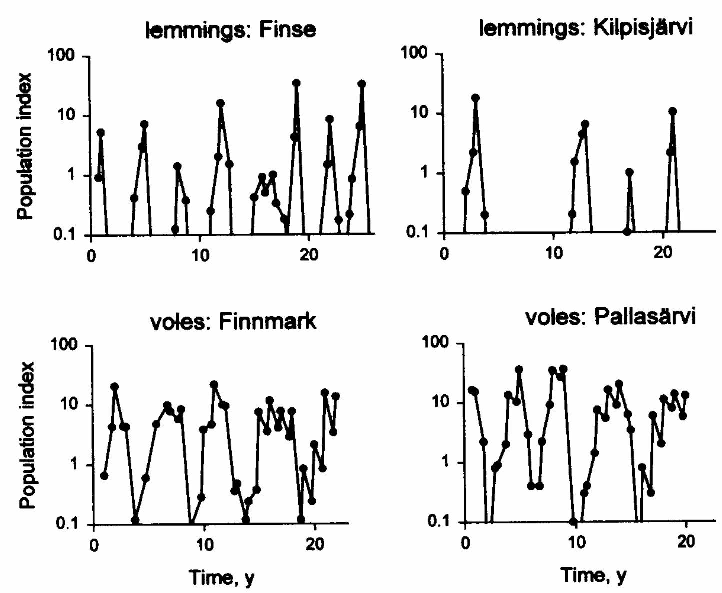
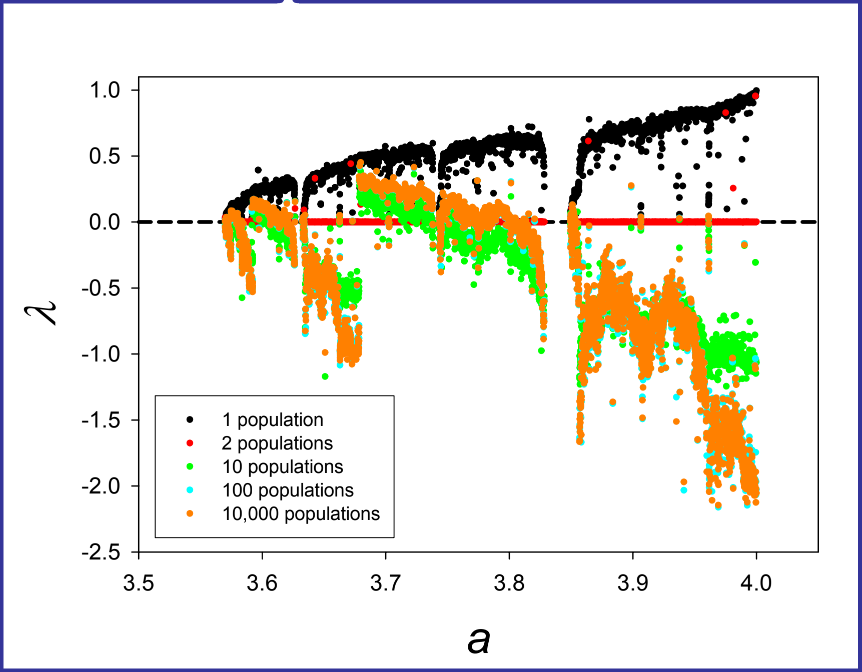


FIGURE 12.5. Population trajectories of lemmings in Finse and Kilpisjärvi; and voles in Finnmark and Pallasjärvi.

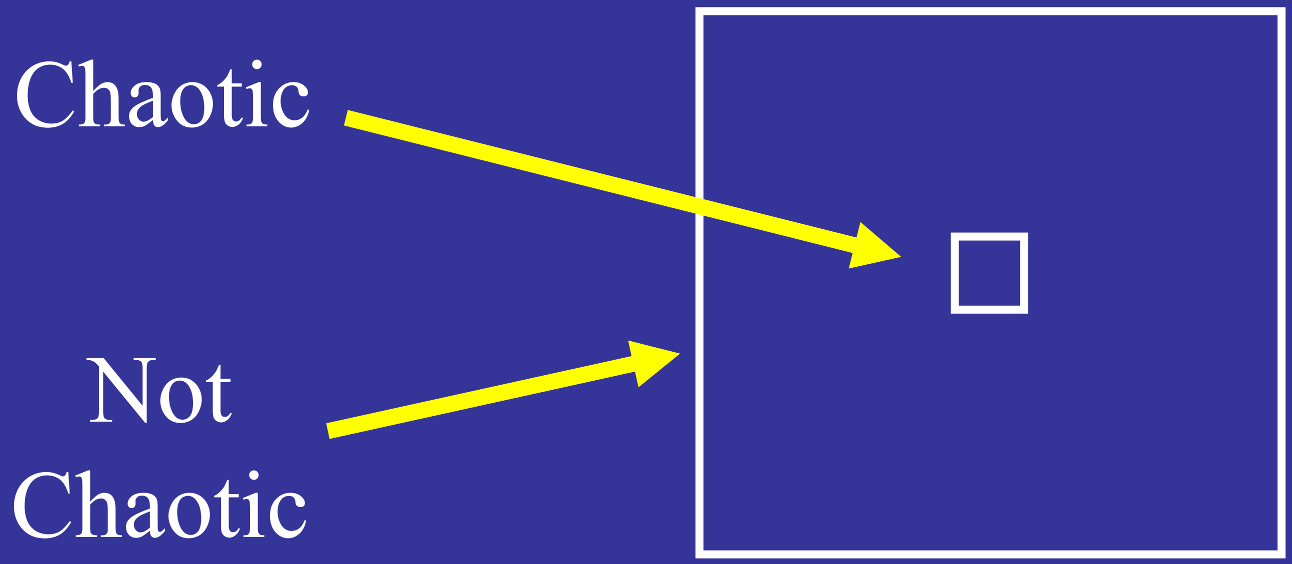
Conclusions (3)

2. If our sampling is large-scale (capturing many small, relatively independent subpopulations, the average of these is occasionally chaotic, but is more likely non-chaotic or "quasi-chaotic."



Conclusions (3)

3. This analysis suggests that the failure to find "**holy chaos,**" especially in rapidly growing insect populations, may be an issue of scale (biologists sampling efforts may be averaging interesting smaller-scale dynamics).



Acknowledgements

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