The Dynamics of Cooperation in Small World Networks

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The Dynamics of Cooperation in Small World Networks

- Introduction
- So, lets say that I have a piano…
- What is a Small World Network?
- Results
- Acknowledgements

So, lets say that I have a piano…
(How the model works)

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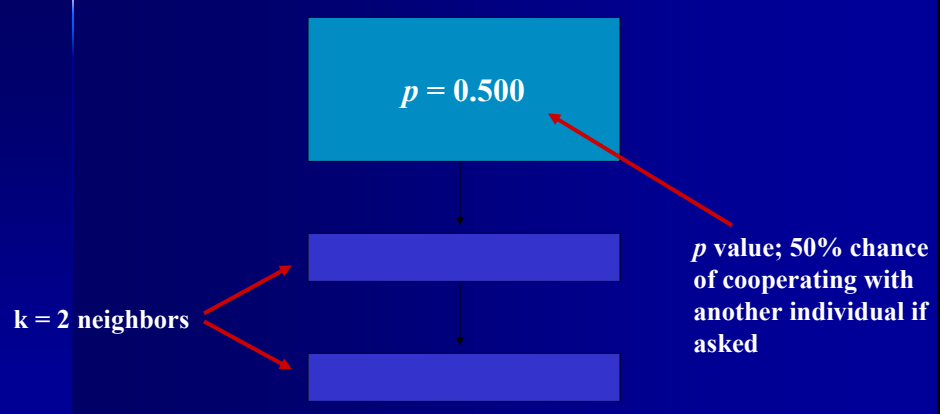
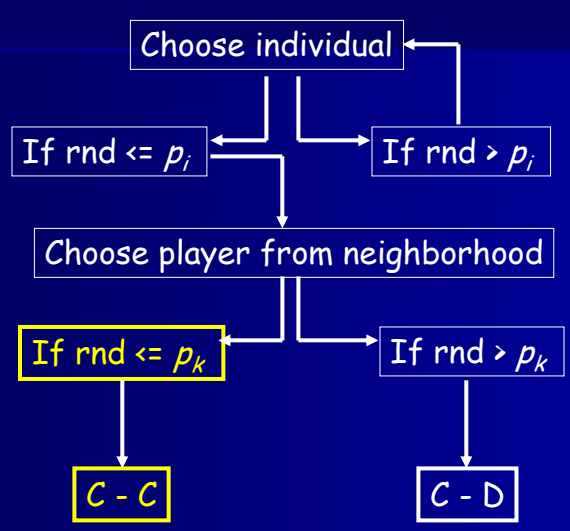
Our Simulated "Individual"



Rules of Engagement

- Model runs in discrete time
- For each time step, we visit every node on the network

Rules of Engagement



Rules of Engagement

- After an interaction, an individual's *p* will go either up or down depending on the whether their neighbour cooperated with them or not.
- This change will be ±ε, where ε is a user-defined variable into the model

What is a Small World Network?

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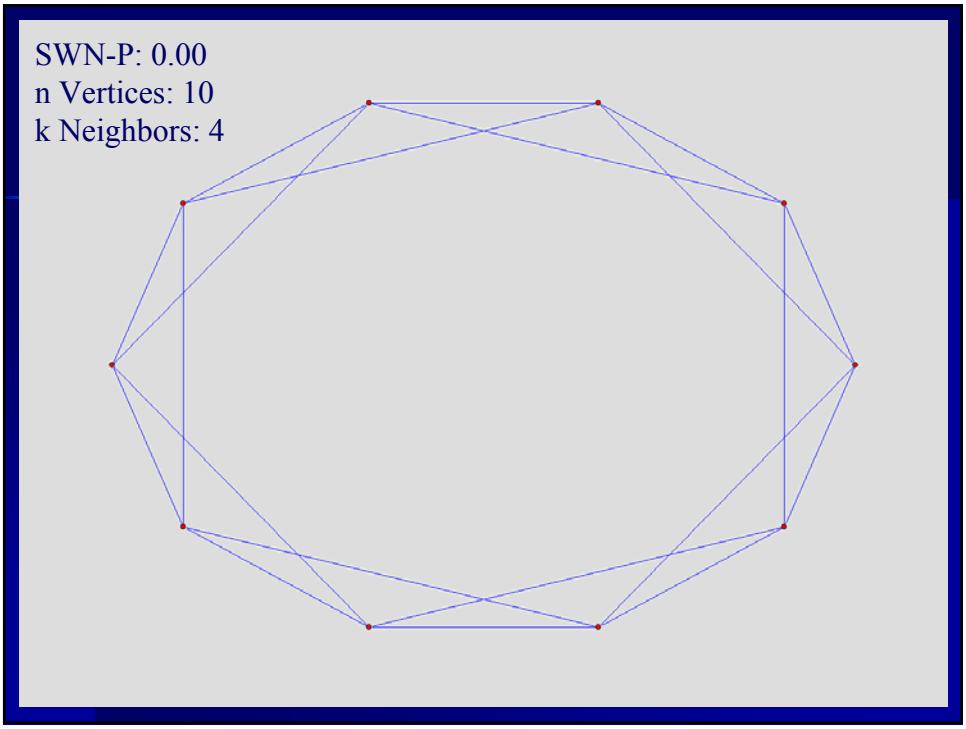
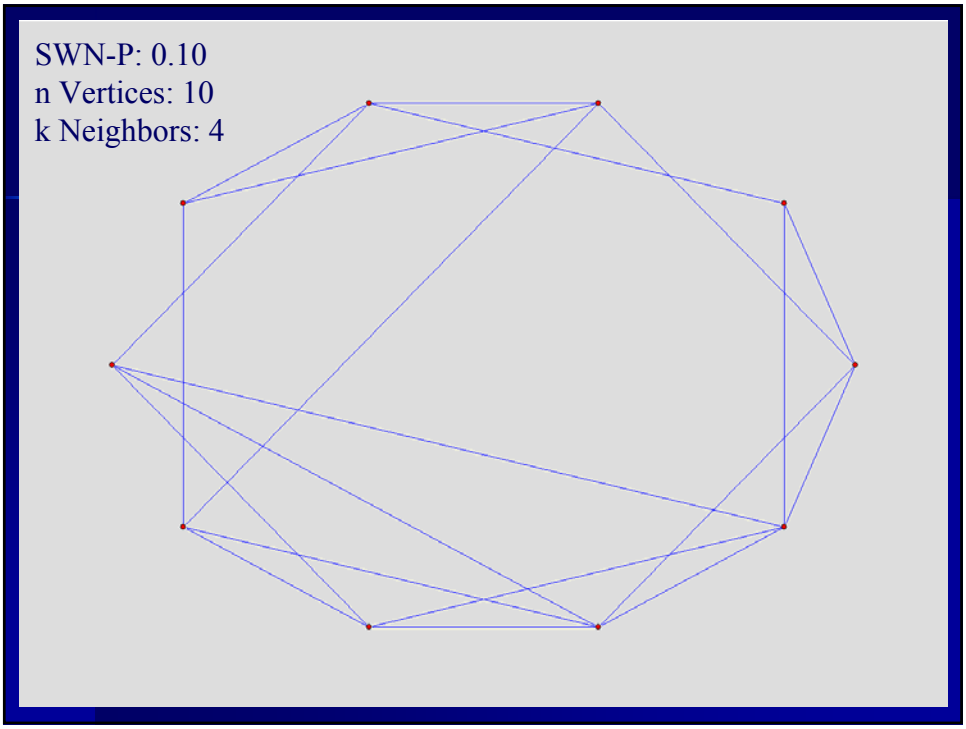
What is a Small World Network?

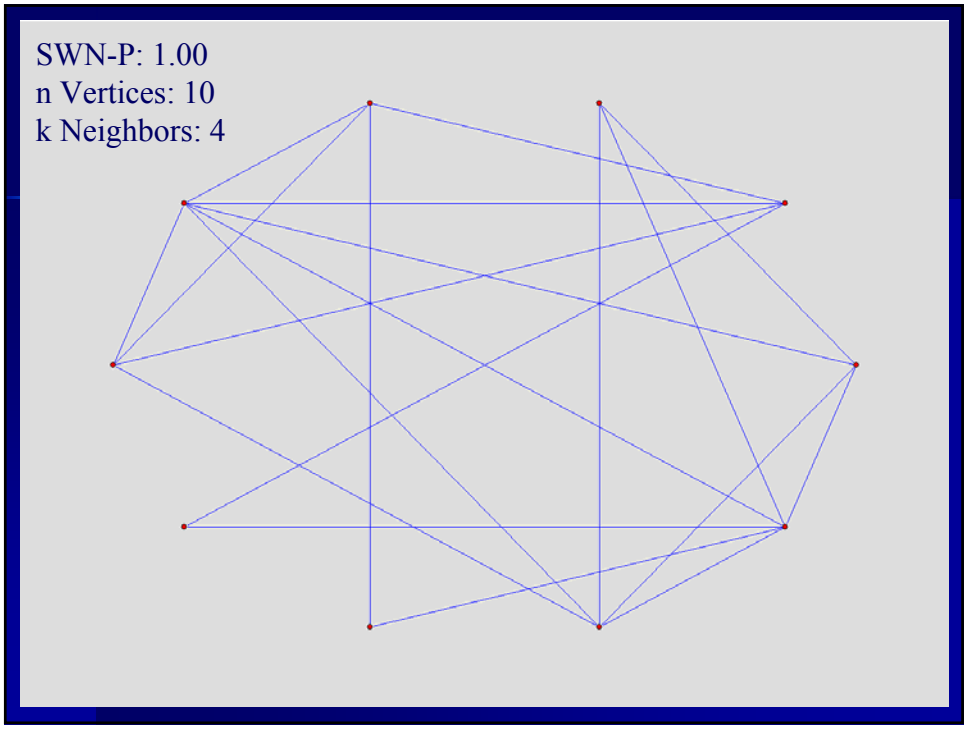
- Think of this as "Six Degrees of Kevin Bacon"; strangers are all connected through mutual acquaintances
- swnP – "Small World Network p"; the probability that an edge from any given vertex in a circulant will be rewired

What is a Small World Network?

- L(p) – "Characteristic path length"; the average of the average path length of every vertex in the network. This is small on a SWN
- C(p) – "Clustering coefficient"; the proportion of the host neighbors that are connected to each other relative to the number of possible connections. This is high on a SWN.


SWN-P: 0.00
n Vertices: 10
k Neighbors: 4


SWN-P: 0.10
n Vertices: 10
k Neighbors: 4



Results

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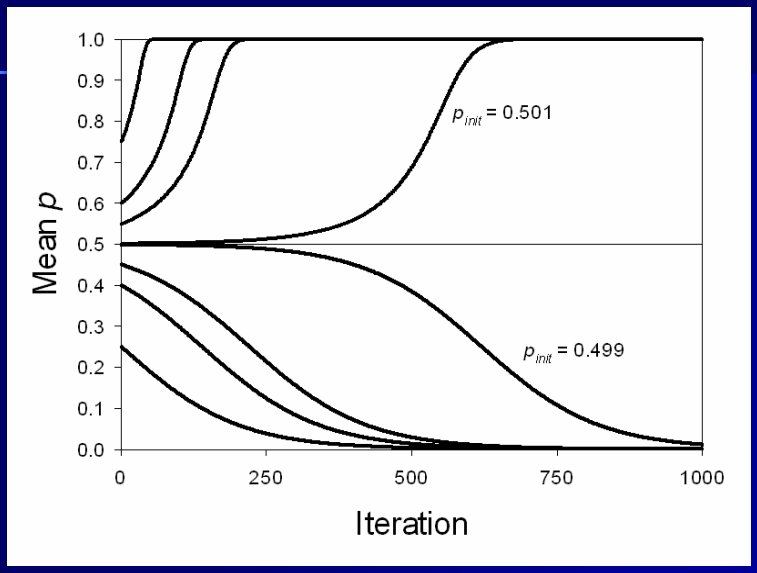
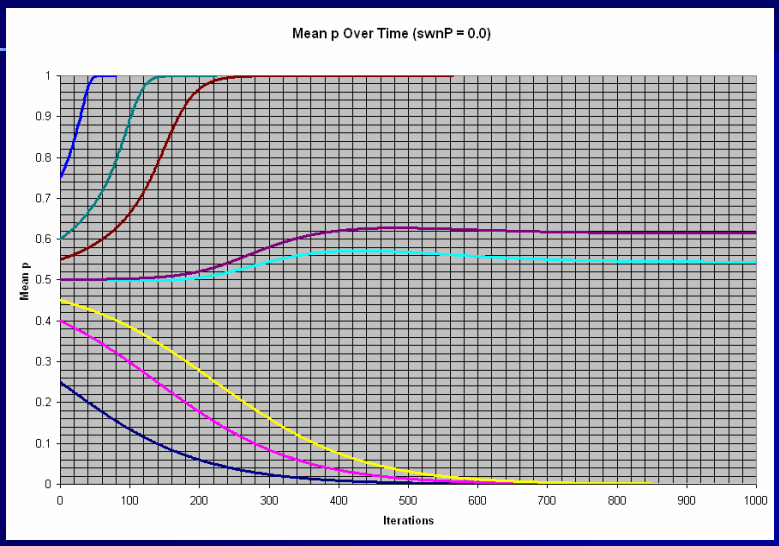
Results

On a lattice, a single defector spreads; simulations go either to global cooperation or defection based on initial *p*



Results

On the SWN, there exists a small area of parameter space where simulations never go to either cooperation or defection. This is around initial *p* = 0.50.



Results

- All simulations had…
  1. k = 8
  2. constant initial *p* distribution
  3. 10000 nodes in community
  4. Epsilon of 0.01
- Simulations without mutation had…
  1. Initial *p* of 0.500
  2. SWN-P values of 0.0, 0.01, 0.05, 0.10, 0.25, 0.50, 1.00
  3. Five replicate simulations

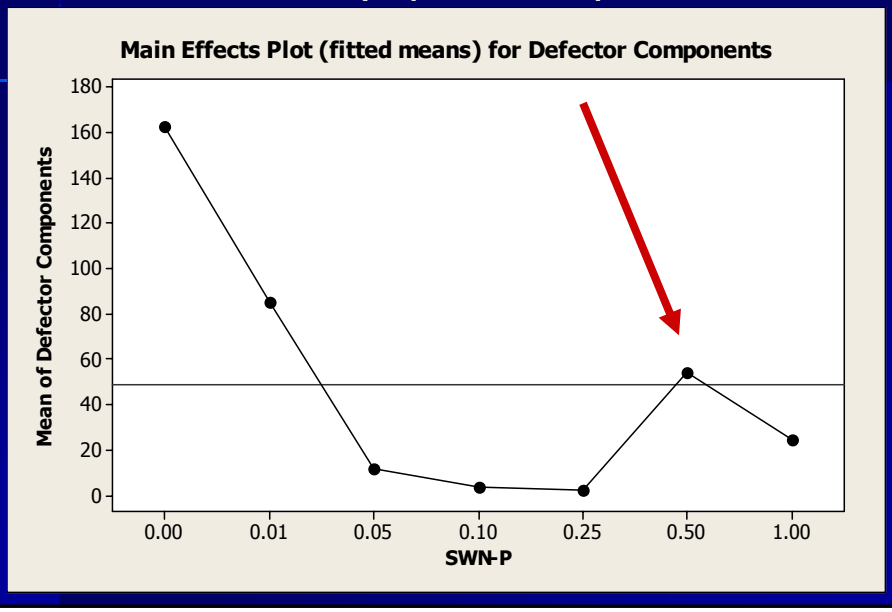
Results

So what exactly are "components"?

- Components are the groups formed by the presence of one or more of the same type of node (e.g. cooperator or defector)
- Larger components (size > 1) are structures of linked individuals of the same type. For example, a series of defecting individuals that are all connected to each other.

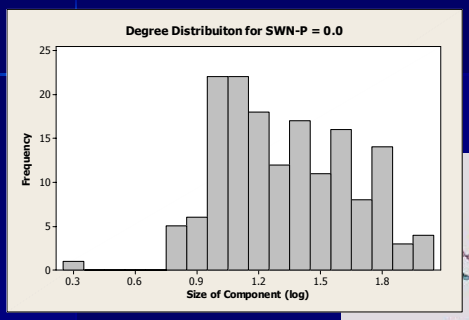
Results

The number of defector components steadily decrease until SWN-P = 0.25, after which they experience a sharp increase

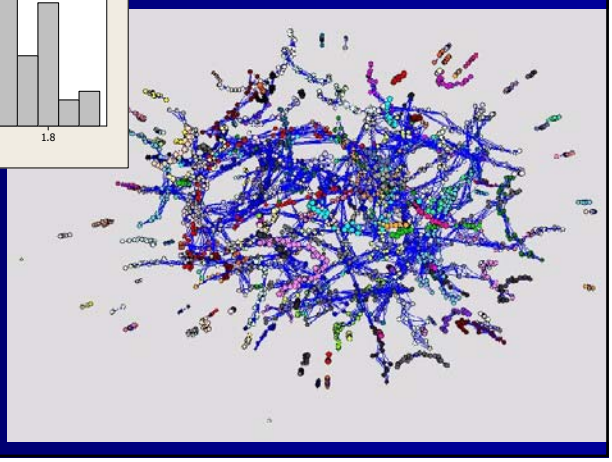


Results

On a circulant, the distribution of component sizes covers a diverse area; there are many components of varying sizes

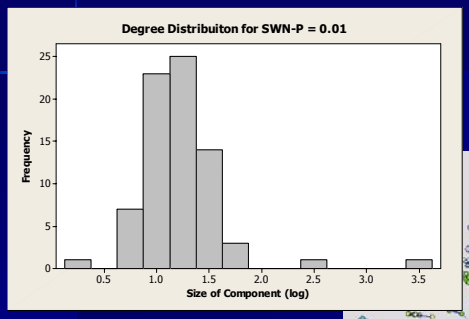


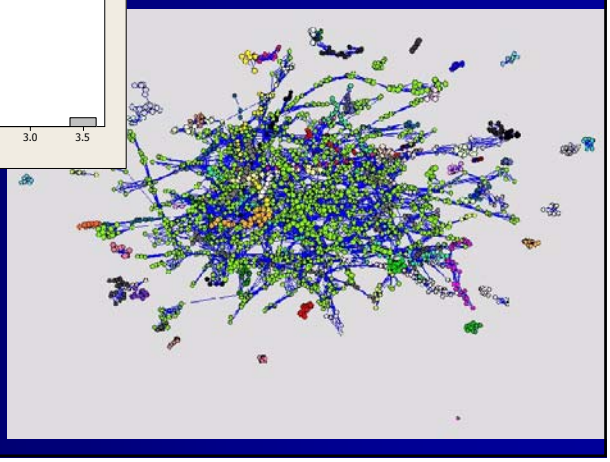
SWN-P = 0.00
Mean k = 8
n = 10000 vertices



Results

When SWN-P = 0.01, the distribution of component sizes is more limited; medium-sized components have merged into a single large component

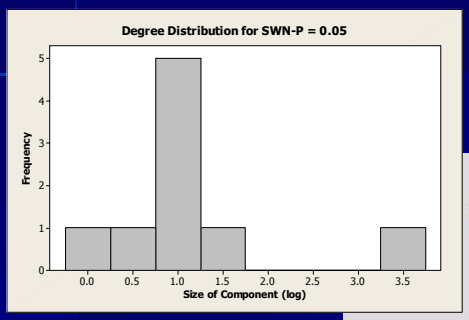


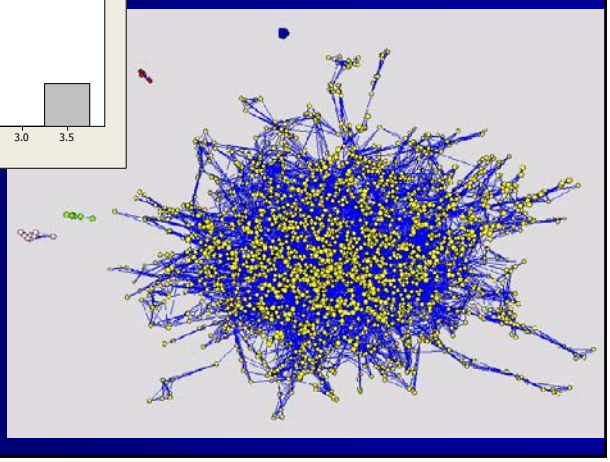


SWN-P = 0.01
Mean k = 8
n = 10000 vertices

Results

When SWN-P = 0.05, the distribution of component sizes is even more extreme; the large component is a few thousand nodes large





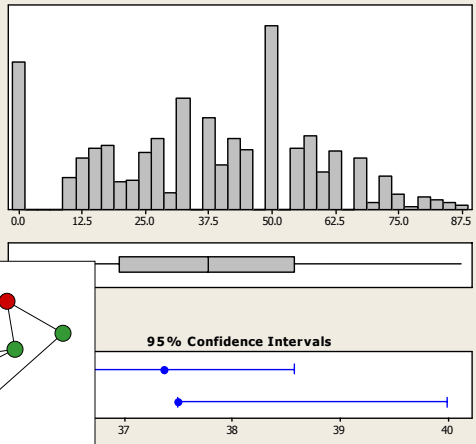
SWN-P = 0.05
Mean k = 8
n = 10000 vertices

Results

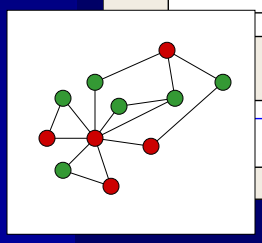
At SWN-P = 0.50, most defecting individuals have less than half of their neighbors as other defectors

Distribution of the Percentage of Defecting Neighbors

SWN-P = 0.5



Anderson-Darling Normality Test	
A-Squared	8.21
P-Value <	0.005
Mean	37.373
StDev	21.348
Variance	455.744
Skewness	-0.073985
Kurtosis	-0.757065
N	1192
Minimum	0.000
1st Quartile	20.000
Median	37.500
3rd Quartile	54.371
Maximum	87.500
95% Confidence Interval for Mean	
36.159	38.586
95% Confidence Interval for Median	
37.500	40.000
95% Confidence Interval for StDev	
20.524	22.241

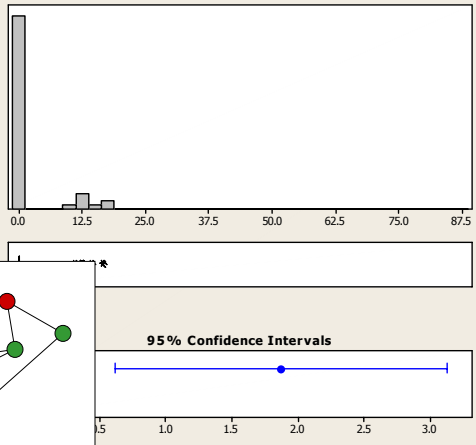


Results

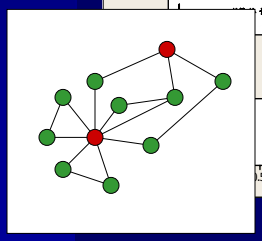
On a random network, most defecting individuals are connected to no other defecting individuals. They are surrounded by cooperators.

Distribution of the Percentage of Defecting Neighbors

SWN-P = 1.0



Anderson-Darling Normality Test	
A-Squared	16.87
P-Value <	0.005
Mean	1.8747
StDev	4.7835
Variance	22.8818
Skewness	2.26464
Kurtosis	3.47530
N	58
Minimum	0.0000
1st Quartile	0.0000
Median	0.0000
3rd Quartile	0.0000
Maximum	16.6667
95% Confidence Interval for Mean	
0.6169	3.1324
95% Confidence Interval for Median	
0.0000	0.0000
95% Confidence Interval for StDev	
4.0440	5.8565

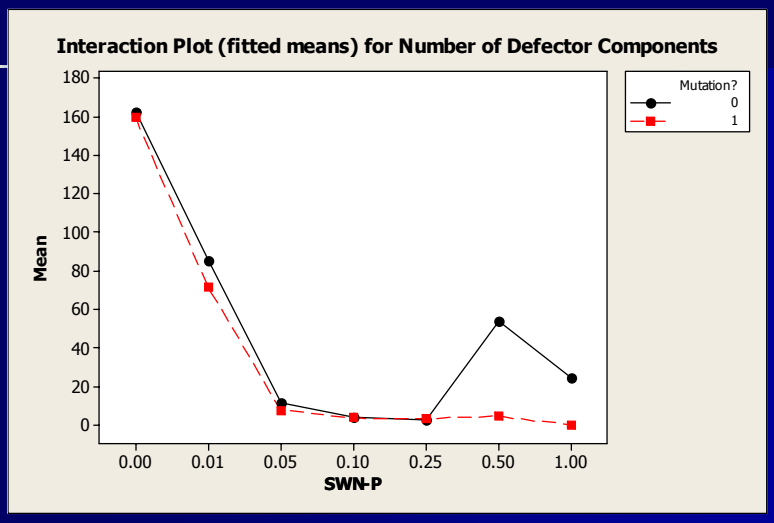


Results

- Simulations with mutation had…
  1. Initial *p* of 0.500
  2. SWN-P values of 0.0, 0.01, 0.05, 0.10, 0.25, 0.50, 1.00
  3. Five replicate simulations
  4. Mutation occurs with a probability of 1/100
  5. In a mutation, an individual's *p* can change by ±0.001.

Results

Mutation has an effect on the number of defector components; many of the isolated defector components are pulled to cooperation

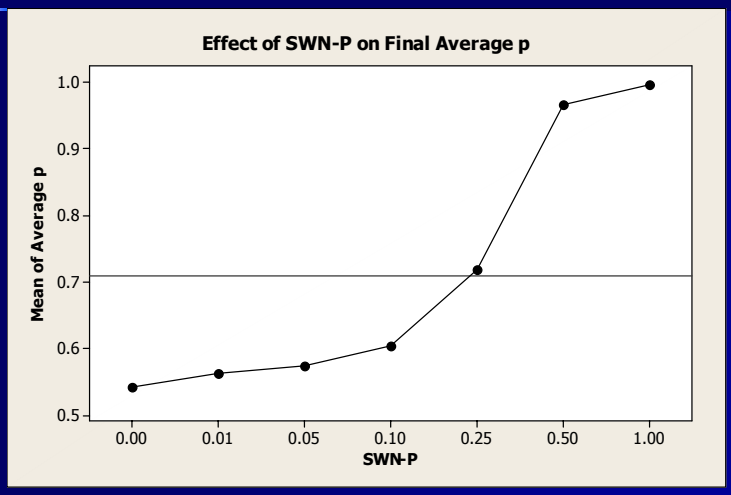


Conclusions

- There are four conclusions from my research to-date.
- Let's briefly summarize them…

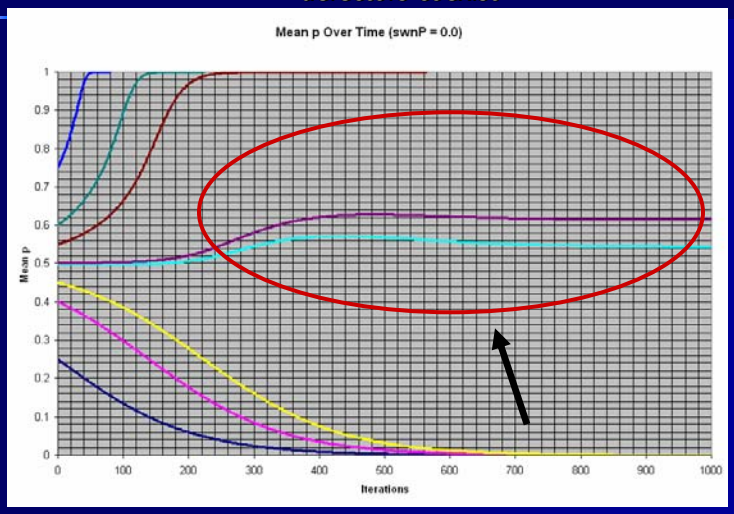
Conclusions

1. Cooperation is more likely on more random networks.



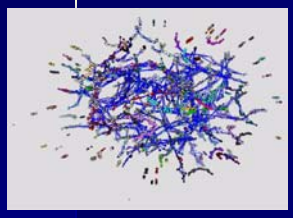
Conclusions

2. In simulations with initial p near 0.5, both cooperators and defectors coexist

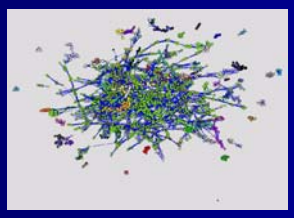


Conclusions

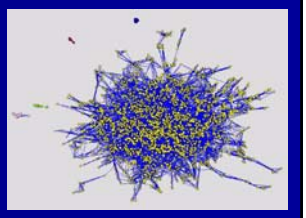
3. Increasing SWN-P, we find that our components merge into two large components of cooperators and defectors



SWN-P = 0.0



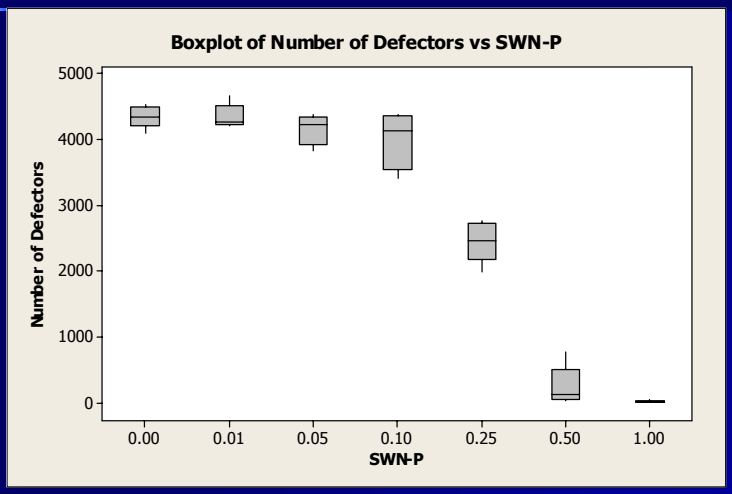
SWN-P = 0.01



SWN-P = 0.05

Conclusions

4. In random networks, there is a negligible amount of defectors in the system



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Muchas Gracias

- Dr. Hartvigsen
- SUNY Geneseo Biomath Program and fellow biomath researchers
- Professor Homa Farian and Matt Haas, of the SUNY Geneseo Distributed Lab
- Rob, for telling me to go into research
- Stacy, for saving me from pointer-error purgatory
- Chris, just because