The Dynamics of Cooperation in Small World Networks

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Introduction

So, lets say that I have a piano...

What is a Small World Network?

Results

Acknowledgements
So, let's say that I have a piano…

(How the model works)

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$p = 0.500$

$p$ value; 50% chance of cooperating with another individual if asked

$k = 2$ neighbors
Rules of Engagement

- Model runs in discrete time
- For each time step, we visit every node on the network
Rules of Engagement

- After an interaction, an individual’s $p$ will go either up or down depending on whether their neighbour cooperated with them or not.
- This change will be $\pm \varepsilon$, where $\varepsilon$ is a user-defined variable into the model.

What is a Small World Network?

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Think of this as “Six Degrees of Kevin Bacon”; strangers are all connected through mutual acquaintances.

- swnP – “Small World Network p”; the probability that an edge from any given vertex in a circulant will be rewired.

What is a Small World Network?

- L(p) – “Characteristic path length”; the average of the average path length of every vertex in the network. This is small on a SWN.
- C(p) – “Clustering coefficient”; the proportion of the host neighbors that are connected to each other relative to the number of possible connections. This is high on a SWN.
SWN-P: 0.00
n Vertices: 10
k Neighbors: 4

SWN-P: 0.10
n Vertices: 10
k Neighbors: 4
Results

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Results

On a lattice, a single defector spreads; simulations go either to global cooperation or defection based on initial $p$.

Results

On the SWN, there exists a small area of parameter space where simulations never go to either cooperation or defection. This is around initial $p = 0.50$. 

![Graph showing mean $\rho$ over iterations for different $p$ values on the SWN.](graph.png)
### Results

- **All simulations had...**
  1. $k = 8$
  2. constant initial $p$ distribution
  3. 10000 nodes in community
  4. Epsilon of 0.01

- **Simulations without mutation had...**
  1. Initial $p$ of 0.500
  2. SWN-P values of 0.0, 0.01, 0.05, 0.10, 0.25, 0.50, 1.00
  3. Five replicate simulations

### Results

**So what exactly are “components”?**

- Components are the groups formed by the presence of one or more of the same type of node (e.g. cooperator or defector)

- Larger components (size > 1) are structures of linked individuals of the same type. For example, a series of defecting individuals that are all connected to each other.
Results
The number of defector components steadily decrease until SWN-P = 0.25, after which they experience a sharp increase.

Main Effects Plot (fitted means) for Defector Components

Results
On a circulant, the distribution of component sizes covers a diverse area; there are many components of varying sizes.

Degree Distribution for SWN-P = 0.0

SWN-P = 0.00
Mean k = 8
n = 10000 vertices
Results

When SWN-P = 0.01, the distribution of component sizes is more limited; medium-sized components have merged into a single large component.

SWN-P = 0.01
Mean k = 8
n = 10000 vertices

Results

When SWN-P = 0.05, the distribution of component sizes is even more extreme; the large component is a few thousand nodes large.

SWN-P = 0.05
Mean k = 8
n = 10000 vertices
**Results**

At SWN-P = 0.50, most defecting individuals have less than half of their neighbors as other defectors.

![Distribution of the Percentage of Defecting Neighbors (SWN-P = 0.5)](image1)

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**Results**

On a random network, most defecting individuals are connected to no other defecting individuals. They are surrounded by cooperators.

![Distribution of the Percentage of Defecting Neighbors (SWN-P = 1.0)](image2)
Simulations with mutation had...

1. Initial $p$ of 0.500
2. SWN-P values of 0.0, 0.01, 0.05, 0.10, 0.25, 0.50, 1.00
3. Five replicate simulations
4. Mutation occurs with a probability of 1/100
5. In a mutation, an individual’s $p$ can change by ±0.001.

Results

Mutation has an effect on the number of defector components; many of the isolated defector components are pulled to cooperation
There are four conclusions from my research to-date.

Let’s briefly summarize them...

1. Cooperation is more likely on more random networks.
Conclusions

2. In simulations with initial $p$ near 0.5, both cooperators and defectors coexist.

Conclusions

3. Increasing SWN-P, we find that our components merge into two large components of cooperators and defectors.
Conclusions

4. In random networks, there is a negligible amount of defectors in the system.

![Boxplot of Number of Defectors vs SWN-P]

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